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Probability analysis of return period of daily maximum rainfall in annual data set of Ludhiana, Punjab

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ABSTRACT

The daily rainfall data of 38 years were collected and one day maximum rainfall was sorted to estimate the probable one day maximum rainfall for different return periods by using probability distribution function. The mean value of annual one day maximum rainfall was found to be 105.9 mm with standard deviation and coefficient of variation in percent and skewness of 64, 0.604 and 2.2 respectively. Three probability distributions such as Log Normal, Gumbel and Log Pearson Type-III distribution had been used to determine the best fit probability distribution that describes the annual one day maximum rainfall by comparing with the Chi-square value. The results revealed that the Log Pearson Type-III distribution was the best fit probability distribution to describe annual one day maximum rainfall. Based on the best fit probability distribution, the maximum of 373.42 mm rainfall could be received with 25 years return period. It could be seen that as the confidence probability increased, the confidence interval also found increased. Further, an increase in return period, T caused the confidence band to spread on. The results from the study could be used to design soil and water conservation structures, irrigation and drainage systems and their managements.

Key words: Chi-square value, Confidence interval, Log pearson type-III distribution, Probability analysis, Rainfall.

INTRODUCTION

Rainfall is one among the main components of hydrological cycle and is considered as principle source of water to the earth. Dependence of Indian agriculture to rainfall is as old as civilization. The success or failure of crops particularly under rainfed condition is closely linked with amount and distribution of rainfall. When the rainfall during a period of year is low or ill distributed, it becomes difficult for the crops raised to meet their ET requirement and that leads to crop failure. On the other hand, if the rainfall is too high as compared to infiltration rate of the soil, it causes higher rate of runoff, resulting in landslides, floods and debris disaster. Hence, knowledge on maximum rainfall distribution over a catchment/watershed is a pre-requisite for proper planning and design of various soil and water conservation structures.

Rainfall data are being analyzed in different ways depending on the problem under consideration. For example, analysis of consecutive days maximum rainfall is more relevant for drainage design of agricultural lands (Bhattacharya and Sarkar, 1982; Upadhaya and Singh, 1998), where as analysis of weekly rainfall data is more useful for planning cropping pattern art its management. The analysis of rainfall data deals with interpreting past record of rainfall events in terms of future probabilities of occurrence. The analysis of rainfall data for computing expected rainfall of a given frequency is commonly done by utilizing different probability distributions. Frequency analysis of rainfall data had been done for different places in India (Jeevrathnam and Jaykumar, 1979; Sharda and Bhushan, 1985;Prakash and Rao, 1986; Aggarwal *et al.*, 1988; Rizvi *et al.*, 2001; Singh, 2001).

Baskar *et al*. (2006) did frequency analysis of consecutive days peak rainfall at Banswara, Rajasthan, India, and found gamma distribution as the best fit as compared to other methods after due testing with Chi-square value. Kwaku and Duke (2007) revealed that the log-normal distribution was the best fit probability distribution tool for analyzing five consecutive days' maximum rainfall in respect of Accra, Ghana.

At present only few studies have been done in India and these studies were mainly carried out to validate the statistical types of probability distribution function, *viz.*, Normal, Log Normal and Gamma. In the present paper, frequency analysis of annual maximum daily rainfall data of Ludhiana, Punjab, was done.

MATERIALS AND METHODS

The analysis was done at Punjab Agricultural University, Ludhiana. The annual maximum daily rainfall

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data for 38 years (1970 to 2007) for Ludhiana station were collected and used for analysis. Annual maximum daily rainfall was sorted out from these data. The statistical behavior of any hydrological series can be described on the basis of certain parameters. Generally, mean, standard deviation, coefficient of variation and coefficient of skewness were taken as measures of variability of hydrological series. All the parameters were used to describe the variability of rainfall in the present study.

From the data, values of one day maximum rainfall were taken for the purpose of study. Return period 'T' was computed using the Weibull's formula as given below:

$$T= (n+1)/m$$

where, 'n' is the total number of years of record and 'm' is the rank of observed rainfall values when arranged in descending order. The probability of exceedence of rainfall values is the reciprocal of the return period.

Frequency analysis: Based on theoretical probability distributions, it could be possible to forecast the incoming rainfall of various magnitudes with different return periods. The probability distributions, most commonly used to estimate the rainfall frequency are Log-Pearson Type-III distribution, Log-normal distribution, Gumbel distribution.

Chow (1964) suggested that rainfall analysis by theoretical probability distributions can be done by using frequency factor 'K' which is based on some statistical parameters. Methods used for assessing probability distribution are as follows:

(i)Log-Pearson Type-III Distribution: In Log Pearson Type-III distribution the value of variate 'X' (rainfall) is transformed to logarithm (base 10). The expected value of rainfall 'R' can be obtained by the following formulae.

$$R = \text{Antilog } X$$

$$\text{Log } X = M + K S$$

where, 'M' is the mean of logarithmic values of observed rainfall and 'S' is the standard deviation of these values. Frequency factor 'K' is taken from Benson, (1968) corresponding to coefficient of skewness of transformed variate.

(ii)Log normal distribution: In Log normal distribution the value of variate 'X' (rainfall) is replaced by its natural logarithm. The expected value of rainfall 'R' can be obtained by following formula:

$$R = \text{Exp}(X) \text{ and}$$

$$\ln X = M\ (1+C_v K)$$

where, 'M' is the mean of natural logarithmic values and 'C_v' is the coefficient of variation of these values. Frequency factor 'K' is taken from Chow, (1964) corresponding to coefficient of variation of transformed variate.

The probability density function of this distribution is

$$p(x) = \frac{1}{\sigma_y e^y \sqrt{2\pi}} e^{-(y-\mu_y)^2/2\sigma_y^2}$$

Where,x = variable;μ= mean value of variable; a ó=standard deviation .

In this distribution mean, mode and median and same. The total area under disatribution is equal to unity.

iii) Gumble distributions

According to Gumbel distribution the expected rainfall 'R' is computed by the following formula

$$R = X_m(1+ C_v K)$$

Where, 'X_m' is the mean of observed rainfall and 'C_v' is the coefficient of variation. Frequency factor 'K' is calculated by the formula given by Gumbel, (1954)

$$X = \text{'}X + K\sigma_X$$

The value of 'K' are computed from their relation

$$K = \frac{\sqrt{6}}{\pi}\left[Y + \text{In}\left(\frac{T}{T-1}\right)\right]$$

This distribution results from any initial distribution of exponential type, which converts to an exponential function, as 'X' increases. The examples of such initial distribution are normal, chi-square and log normal distributions. The probability density function of this distribution is

$$p(x) = \frac{1}{c} e^{-(a+x)/c - e^{-(a+x)/c}}$$

with -∞<x∞ <, where 'x' is the variate and 'a' and 'c' are parameter.

The parameter have been evaluated by the method of moment as:

$$a=yc- \mu$$

$$c= \frac{\sqrt{6}}{\pi}\sigma_x$$

where, y = 0.5772 = Euler's Constant; μ is the mean; σ_x is the standard deviation

Testing the goodness of fit of probability distribution of different methods used: For the purpose of prediction, it is usually required to understand the shape of the underlying distribution of the population. To determine the underlying distribution, it is a common practice to fit the observed distribution to a theoretical distribution. This is done by comparing the observed frequencies in the data to the expected frequencies of the theoretical distribution since certain types of variables follow specific distribution (Tilahun, 2006).

One of the most commonly used tests for testing frequency distribution is the chi-square test (Haan, 1977). The test compares the actual number of observations and the expected number of observations (expected values are calculated based on the distribution under consideration) that fall in the class intervals.

The Chi-square test statistic is computed from the following relationship

$$x^2 = \sum_{i=1}^{m} (Oi - Ei)\,2/Ei$$

Where, O_i is the observed and E_i the expected rainfall. The distribution of $c^2$is the chi-square distribution with n-m-1 degree of freedom.

The probability density functions, Log-Pearson Type-III distribution, lognormal distribution, Gumbel distribution were used for analysis and compared with the Weibull's method for deciding the best fitting distribution. While comparing the probable rainfall at different levels, the Weibull's method was considered as nearly equal to observed distribution.

The distribution that gives the smallest Chi square value (Agarwal *et al.,* 1988) was selected are recommended as best fit probability distribution function for the study area.

Since the value of the variate for the given return period, determined from log pearson type-III method can have errors due to the limited sample data used, an estimate of the confidence limits of the estimate is desirable. The confidence interval indicates the limits about the calculated value between which the true value can be said to lie with a specified probability based on sampling errors only**.** The size of confidenceinterval depends on the confidence level β.

Corresponding to the confidence level β a significance level α, given by

$$\alpha = 1- \beta/2$$

For estimating the event magnitude for return period T, the upper limit $U_{T,\alpha}$ and lower limit $L_{T,\alpha}$ may be specified by adjustment of the frequency factor equation:

$$U_{T,\alpha} = y + s_y K^U_{T,\alpha}$$
$$L_{T,\alpha} = y + s_y K^L_{T,\alpha}$$

Where $K^U_{T,\alpha}$ and $K^L_{T,\alpha}$ are upper and lower confidence limit factors. s_y is coefficient of skewness. The values for these factors are given by the following formulas:

$$K^U_{T,a} = \frac{K+\sqrt{K^2-ab}}{}$$
$$K^L_{T,a} = \frac{K-\sqrt{K^2-ab}}{a}$$

In which,

$$a = 1- Z^2/2(n-1)$$
and $$b = K\ (\text{for } T) - Z^2/n$$

The quantity Z is the standard normal variable with exceedence probability α.

RESULTS AND DISCUSSION

Statistical parameters: The average, standard deviation, coefficient of variation and skewness of annual one day daily maximum rainfall for 38 years is given in Table 1. The sestatistical parameters can be used to find the estimated one day maximum rainfall from different probability distribution functions.

The expected annual one day maximum rainfall for different probability distributions such as Log Normal, Gumbel's and Log Pearson Type-III were calculated and presented in Table 2 for different return periods. The expected annual one day maximum rainfall for different return period are graphically represented in Figure1. From the figure, it is observed that the estimated annual one day maximum rainfall for different probability distributions follow the same trend of observed rainfall for different return periods.

The analysis of daily rainfall data revealed that the Log-pearson type-III distribution was the best-fit distribution with minimum variance (217.46) among the various probability distribution functions considered (Log normal distributions, Gumbel distribution) in comparison with the Weibull's observed distribution (Table 2). The second and third best-fit distributions were Gumbel and Log normal distributions with the chi-square values of 236.69 and 502.36 respectively. According to this distribution, in a day, the maximum rainfall of 373.42 mm rainfall could be received with 25 year return period.

Regression model was developed from the observed annual one day maximum rainfall against different return periods by using Weibull's method. The trend analysis (Fig 1.) for prediction of one day maximum rainfall for different return periods was carried out and it is found that the exponential trendline gave better coefficient of determination [$(R^2) = 0.979$] and the equation is: $Y = 33.36x^{1.217}$, where x is the chi square value of weibull's method corresponding to return period.

Reliability of analysis: The 95 per cent confidence limit for one day maximum rainfall was estimated using Log pearson type-III distribution are presented in Table 3 and plotted in Fig. 2.

The 95 per cent confidence limit for one day maximum rainfall for different return periods of 2, 5, 10, 15, 20 and 25 years were 70.79 and 116.94(91.2) mm, 134.89

TABLE 1: Statistics of maximum one-day rainfall at Ludhiana station (1970-2007).

Parameter	Values
Average one day maximum rainfall (mm).	105.9
Standard deviation (mm).	64
Coefficient of variation (C_v).	0.604
Coefficient of skewness (C_s).	2.2

and 247.40 (177.82) mm, 186.20 and 377.57(252.87) mm, 211.3 and 442.58 (293.05) mm, 238.17 and 516.41(333.23) mm, 264.66 and 610.94(373.42) mm respectively.

From Fig.2 the result shows that the maximum rainfall value was within the confidence limit. It means that the good fit distribution is reliable.

TABLE 2: Expected rainfall and Chi-square values for different probability distributions function at different return period for different distributions

	Expected rainfall (mm)				$(O-E)^2/E$		
Return period (T), year	Log-Pearson type-III	Log Normal	Gumbel	Weibull (O)	Log- Pearson type-III	Log Normal	Gumbel
1	15.38	31.27	20.74	29.30	12.59	0.12	3.53
2	91.20	92.94	95.39	89.00	0.053	0.16	0.42
3	120.00	108.65	114.27	102.00	2.70	0.42	1.31
4	148.70	124.34	133.15	112.30	8.99	1.16	3.26
5	177.80	140.07	152.05	125.20	15.56	1.57	4.74
6	192.80	144.89	159.55	131.40	19.55	1.25	4.96
7	207.84	149.71	167.05	147.40	17.57	0.03	2.31
8	222.85	154.53	174.55	173.60	10.88	2.35	0.005
9	237.86	159.35	182.05	190.70	9.35	6.16	0.41
10	252.87	164.2	189.59	205.90	8.72	10.59	1.4
11	260.90	167.59	192.73	216.51	7.55	14.27	2.93
12	268.90	170.98	195.87	227.13	6.49	18.42	4.97
13	276.90	174.37	199.01	237.85	5.52	23.07	7.56
14	285.00	177.76	202.15	245.35	5.53	25.66	9.21
15	293.05	181.15	205.29	252.97	5.5	28.41	11.04
16	301	184.54	208.43	260.50	5.44	31.26	13
17	309.10	187.93	211.57	268.00	5.46	34.11	15.05
18	317.10	191.32	214.71	275.63	5.43	37.12	17.26
19	325.10	194.71	217.85	283.22	5.4	40.21	19.6
20	333.23	198.1	220.99	288.32	6.05	41.07	20.5
21	341.26	201.49	224.13	290.00	7.69	38.88	19.35
22	349.30	204.88	227.27	293.51	8.91	38.67	19.3
23	357.30	208.27	230.41	296.10	10.48	37.03	18.72
24	365.37	211.66	233.55	298.74	12.16	35.79	18.17
25	373.42	215.14	236.69	301.40	13.89	34.58	17.69
				Total	217.46	502.36	236.69

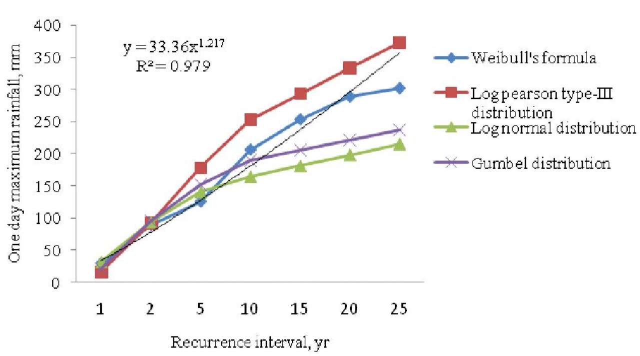

FIG 1: Estimated annual one day maximum rainfall for different return period

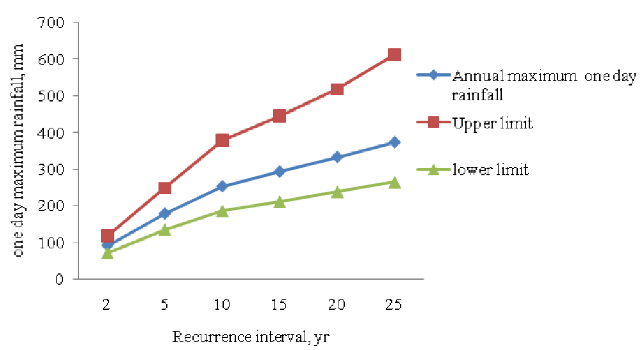

FIG 2: Confidence limit band for maximum rainfall by Log pearson type-III distribution.

TABLE 3: Confidence limits for one day maximum rainfall estimated using Log pearson type-III distribution at 95% confidence limit.

T, yr	b	K^U	K^L	95% Confidence limit	
				Upper limit	Lower limit
2	-0.100	0.300	-0.352	116.94	70.79
5	0.591	1.270	0.485	247.40	134.89
10	1.579	1.819	0.919	377.57	186.20
15	2.045	2.025	1.065	442.58	211.30
20	2.567	2.275	1.220	516.41	238.17
25	3.146	2.444	1.357	610.94	264.66

CONCLUSIONS

Rainfall is highly variable in space and time and subject to variability with natural and anthropogenic causes. The frequency analysis of annual one day maximum rainfall for identifying the best fit probability distribution was done by using three probability distributions viz. Log Normal, Gumbel's and Log Pearson Type-III and selected the best one by using Chi-square goodness of fit test. The results of the study revealed that the average value of annual one day maximum rainfall was 105.9 mm with standard deviation and coefficient of variation of 64 and 0.604, respectively. The coefficient of skewness was observed to be 2.2.

It was observed that Log Pearson type-III distribution was found to be the best method among the three used based on the testing through Chi-square test. For a recurrence interval of 25 years, the annual one day maximum rainfall was 373.42 mm. Regression model for annual one day maximum rainfall was developed by using Weibull's method to predict the rainfall for different return periods. It could be seen that an increase in return period, T caused the confidence band to spread on.

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