BKAP-630
[1-4]

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,YxkfjFke lp dk mi;kx djd xj lkekU; çfrfØ;k d lkFk feJ.k ç;kxk d fy, lrlr Mh&v,IVhey vfHkdYiukvk dk fuek.k

jkgy cutkh[1]] lhek tXxh[2]] ,Ynk oxhl[3]] vi.kk Hkkfed[4]] fluh oxhl[1]] vfufUnrk nÙkk[1]] 'orkd yky[1]

lkjk'k

feJ.k ç;kx lx.kdh; lrg fMtkbu dh ,d [kkl J.kh gkr g] tgk nk ;k vf/kd ?kVdk dk feykdj ,d mRikn çkIr gkrk g] ft l rRok ;k feJ.k ?kVdk dgk tkrk gA feJ.k ç;kxk d ekey e] vDlj çkIr çfrfØ;k x.kkRed gkrh gA xj&lk/kkj.k çfrfØ;k d ekey e] ,d y,ftfLVd e,My lVvi dk mi;kx fd;k tkrk gA bl v/;;u e] geu y,ftfLVd e,My dk mi;kx fd;k g vkj [kkt ,Yxkfjne fodflr djd feJ.k ç;kxk d fy, lrlr Mh&v,fIVey fMtkbu çkIr fd, gA Mh&v,fIVeyrk ekunM dh ekU;rk d fy, geu tujy bfDooyl fl)kr dk mi;kx fd;k gA y,ftfLVd e,My d ek/;e l LFkkfud Mh&v,fIVey fMtkbu r;kj dju d fy, =fV;k dk lkekU; forj.k ugh fd;k tkrk gA ;g çfØ;k vU; xj&jf[kd e,Myk d fy, Hkh leku gkrh g] yfdu =fV forj.k /kkj.kkvk d dkj.k fQ'kj lpuk efVDl ¼FIM½ e ifjoru gkrk gA

"kCn dth% mEehnokj lV] Mh&v,fIVekfyVh] fQ'kj lpuk efVDl] tujy bfDooyU l F;kje] y,ftfLVd e,My] feJ.k ç;kx] l'kkf/kr QMkjko ,Dlpt ,YxkfjFke] xj lkekU; çfrfØ;kA

Construction of Saturated D-optimal Designs for Mixture Experiments with a Non Normal Response using an Algorithmic Search

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ABSTRACT

Background: Mixture experiments belong to the response surface design category, involving the combination of multiple components to create a product. These products are commonly encountered in daily life. In some cases, mixture experiments yield qualitative responses, such as taste in a fruit punch. Qualitative variables often deviate from a normal distribution.

Methods: To address non-normal responses, a generalized linear model, specifically the logistic model, is employed. This study utilizes logistic models and develops suitable search algorithms to obtain saturated D-optimal designs for mixture experiments. The validation of D-optimality criteria is based on the General Equivalence Theorem.

Result: For generating locally D-optimal designs, the logistic model is utilized considering non-normally distributed errors. While the procedure remains the same for other nonlinear models, the assumptions regarding error distribution impact the Fisher information matrix (FIM).

Key words: Candidate set, D-optimality, Fisher information matrix, General equivalence theorem, Logistic Models, Mixture experiments, Modified Fedorov exchange algorithm, Non Normal Response.

çLrkouk

,d ç;kx ftl lkf[;dh; fl)krk dk ikyu djr g, 'kkyhurk l r;kj fd;k x;k g] u doy ç;kx drk d ç'u dk lVhd mÙkj çnku djrk g cfYd miyC/k lk/kuk d b"Vre mi;kx e Hkh enn djrk gA feJ.k ,d ,lk mRikn g tk nk ;k nk l vf/kd ?kVdk dk feykdj çkIr fd;k tkrk g ftl vo;o dgk tkrk gA ,l mRikn jkstejk dh ftnxh e ik, tkr g ¼d,uy] 2002½ tl fd%

¼1½ dd r;kj djuk lkexh n/k] phuh] vkVk] v.M] cfdx ikmMj vkfnA

¼2½ dØhV fuek.k jr] ikuh vkj ,d ;k ,d l vf/kd çdkj d lheV dk l;ktuA

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¼3½ ÝV LDok'k/ip/te r;kj djuk% fofHkUu ek=k e Qyk d jl dk mi;kx djd r;kj fd;k tkrk g phuh] ued vkfn d lkFkA

¼4½ QkVkxkfQd fQYe% flYoj gykbM] nk LVcykbtl d vyx&vyx vuikr l çklr nk ;Xed foyk;d vkj ,d ;Xed l r;kj fd;k tkrk gA

¼5½ rEckd d feJ.k% çlL—r rEckd] ¶y&D;kM rEckd] rfd'k lfEeJ.k l feJ.k çklr fd;k tkrk gA

—f"k ç;kxk d ekey e feJ.k ç;kx dh fLFkfr] ykx buiV d db lkr gk ldr g ;k buiV dk vyx&vyx Qly fodkl pj.kk e foHkkftr fd;k tk ldrk g rkfd Qly ij ykx dy ek=k fLFkj jgA mnkgj.k d fy,] Qyk vkj ckxokuh çk|kfxdh foHkkx] Hkkjrh; —f"k vuql/kku lLFkku] ub fnYyh e ,d ç;kx fd;k x;k Fkk fofHkUu vuikrk e fefJr Qyk d jl vkoyk ¼Emblica officinalis½ dk fefJr dj u dh 0;ogk;rk dk v/;;u dju d fy,A 0%100; 5%95; 10%90; 15%85; 20%80; 25%75; 50%50; 75%25; 80%20; 85%15; 90%10; 95%5; 100%0; vuikr fy, x,] i; inkFkk dh lx/k] Lokn vkj ik"kd rRok e l/kkj d fy, jMhV lo ¼vkjVh,l½ i; inkFkk dh r;kjh vkj ekudhdj.k d fy,A ,d feJ.k ç;kx dk fuji{k dgk tkrk g ;fn feJ.k d x.k doy vo;ok d vuikr l fu/kkfjr gkrk g u fd feJ.k d ek=k lA

feJ.k vfHkdYiuk;

eku y fd ,d feJ.k q lkexh dk feyk dj çklr fd;k x;k g] vkj eku y] $x = [x_1, x_2, ..., x_q]$ feJ.k d D; vo;ok d vuikr dk oDVj g] vkj y¼x½ çfrfØ;k dk lxr ek/; g] ;gk dkjd LFkku dk T;kferh; :i l ¼q-1½ vk;keh flEyDl Sq }kjk n'kk;k tk ldrk g ¼Scheffé 1958]63½A

$$S_q = \{x = [x_1, x_2, ...x_q]; 0 \le x_i \le 1, \sum_{i=1}^{q} x_i = 1 \forall i = 1,2,...,q\}$$

flEiyDl dk T;kferh; vkdkj lkexh dh l[;k e ifjoru d lkFk cny tk,xk ;kuh q¾3 d fy, ;g ,d leckg f=Hkt g] q¾4 d fy, ;g ,d VVkgM,u g tlk fd fp= 1 e n'kk;k x;k gA

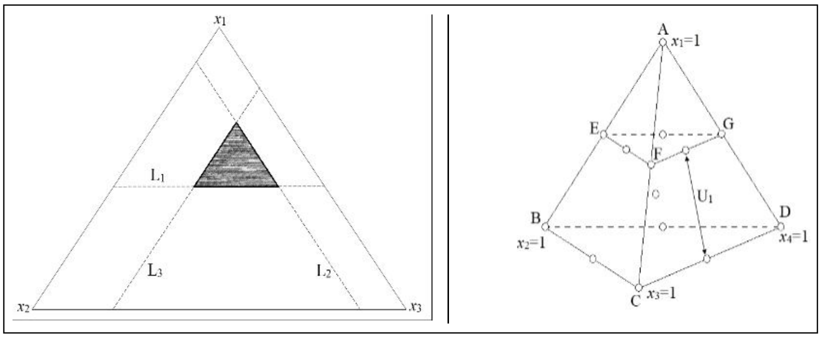
xj&lkekU; çfrfØ;k,

feJ.k ç;kxk d ekey e vDlj ,lh fLFkfr;k gkrh g tgk çklr çfrfØ;k x.kkRed ç—fr e gkrh gA mnkgj.k d fy,] ÝV ip d ekey e çklr çfrfØ;k Lokn gk ldrh g tk ,d x.kkRed x.k gA blh rjg] ,d dd d fy, çfrfØ;k dd dh cukoV gk ldrh g tk ,d x.kkRed x.k Hkh gA x.kkRed çfrfØ;k d fy,] pj vkerkj ij lkekU; forj.k dk ikyu ugh djr gA lkekU;h—r xj&lkekU; çfrfØ;k oky pj d ekey e tujykbtM jf[kd e,My lV vidk mi;kx fd;k tkrk gA ;gk] bl v/;;u e y,ftfLVd e,My dk mi;kx fd;k x;k g vkj y,ftfLVd e,My dk mi;kx djd feJ.k ç;kxk d fy, vfHkdYiuk; çklr fd, x, gA y,ftfLVd e,My ij fopkj fd;k x;k g tgk LFkkuh; :i l Mh&vi,IVhey vfHkdYiuk; r;kj dju d fy, =fV;k lkekU; :i l forfjr ugh gkr gA çfØ;k vj[kh; e,My d leku gh jgrh g] ysfdu =fV forj.k /kkj.kkvk d dkj.k FIM cny tkrk gA

y,ftfLVd e,My

y,ftfLVd e,My tfod vkj lkekftd foKkuk e lcl 0;kid :i l v/;;u fd, x, xj&jf[kd e,My e l ,d gA ;g buiV pj vkj ckbujh/ v,fMuy çfrfØ;k d chp xj&jf[kd lc/k LFkkfir djrk gA

$$P(Y=1) = \frac{e^{\gamma}}{1+e^{\gamma}} = \frac{1}{1+e^{-\gamma}}, Y = 0,1$$



fp= 1% q¾3 ,o q¾4 d fy, flEIyDl dk T;kferh; çfrfuf/kRoA tgk] γ jf[kd çfMDVj ;k 0;k[;kRed pj χ dk jf[kd QD'ku g] ,o θ vKkr ijkehVj g] rk y,ftfLVd e,My dk :i γ dh iln ij fuHkj djrk gA lcl ljy e,My ,d 0;k[;kRed pj ,o nk ekinMk d :i e ifjHkkf"kr fd;k x;k g%

$$\gamma(\chi, \theta) = \beta_0 + \beta_1\chi_1$$

ifjHkkf"kr fyfu;j çfMDVj d lkFk y,ftfLVd e,My d fy, ,d vfHkdYiuk dk FIM fuEufyf[kr l= }kjk n'kk;k tk ldrk g%

$$M(\xi, \theta) = \frac{1}{n}\sum_{i=1}^{n} \frac{e^{-(\beta 0 + \beta 1\chi 1i)}}{(1 + e^{-(\beta 0 + \beta 1\chi 1i)})^2} \begin{bmatrix} 1 & \chi_{1i} \\ \chi_{1i} & \chi^2_{1i} \end{bmatrix} \theta = \theta_0$$

feJ.k ç;kxk e vfHkdYiuk fuek.k d fy, ,Yxkfjne dk mi;kx dju d db Qk;n g tl fd%

- Û vfHkdYiuk d fy, juk dh l[;k puu e vf/kd Lor=rkA
- Û feJ.k ç;kxk e lkexh d fy, vuikr dk p;u dju e vf/kd Lor=rkA
- Û fofHkUu e,Myk dh fQfVx d fy, vfHkdYiuk fuek.k e vf/kd Lor=rkA
- Û vfHkdYiuk ekunMk dk ijk dju d fy, vfHkdYiuk d fuek.k e vf/kd Lor=rkA

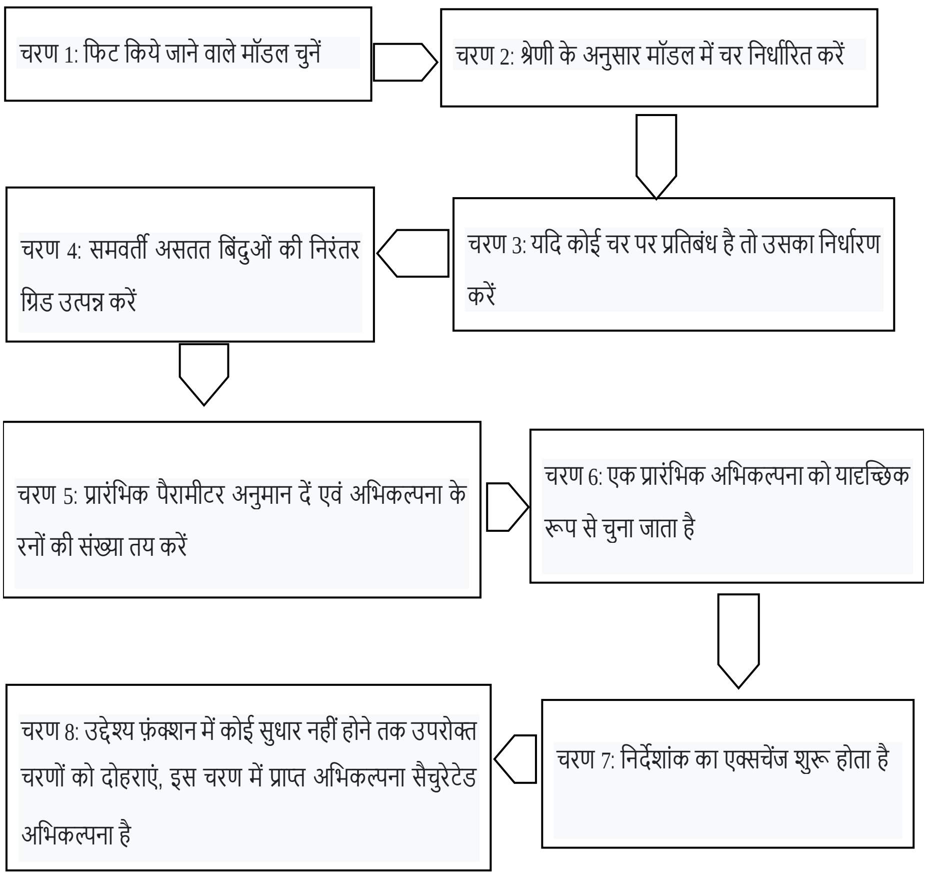
lkfgR; e igy l gh dN ,Yxkfjne miyC/k g ftudk mi;kx igy fMtkbu fuek.k d fy, fd;k x;k gA MVeDl ,YxkfjFke ¼fe'ky] 1974½] d ,y&,Dlpt ,YxkfjFke ¼,Vfdlu vkj Mkuo] 1989½] XVERT ,YxkfjFke ¼Luh ,M ekdokM] 1974½] XVERT 1-0 ,YxkfjFke ¼fuxe] 1983½A xj&jf[kd e,My e v,lVhey vfHkdYiuk fl)kr d fy, miyC/k vf/kdk'k lkfgR; doy ,d 0;k[;kRed pj ij dfær gA ;|fi l)kfrd ifj.kke vkj ch txf.krh; lek/kku [kkstuk pukrhi.k gk ldrk g] db 0;k[;kRed pj okyk e,My d ekey e budk foLrkj djuk ef'dy gA bl ekey e ,Yxkfjne vkj áfjfLVDl dk mi;kx djuk vf/kd mi;kxh gA

i)fr

,d lpjVM vfHkdYiuk dk vFk g fd e,My e vuekfur ijkehVj dh l[;k d cjkcj vfHkdYiuk dk ju gkrk gA fodflr ,YxkfjFe dk o.ku ljy ¶ykopkV ¼fp= 2½ e dN bl çdkj gA

$$\phi_i = \theta_1 x_{i1} + \theta_2 x_{2i} + \theta_3 x_{i1} x_{2i}$$

Åij mfYyf[kr y,ftfLVd lehdj.k nk feJ.k ?kVdk d lkFk ,d bVjD'ku e,My dk çfrfuf/kRo djrk gA



fp= 2% fodflr ,YxkfjFe dk ljy ¶ykpkV }kjk o.ku eku yr gA

rkfydk 1% bVjD'ku d lkFk nk feJ.k ?kVdk d lkFk y,ftfLVd e,My d rgr feJ.k ç;kx d fy, Mh&v,lVhey lrlr vfHkdYiukA

e,My	$\phi_i = \theta_1 x_{1i} + \theta_2 x_{2i} + \theta_3 x_{1i} x_{2i}$		
bUØheV	dfMMV lV dk vkdkj	Mh&v,lVhey vfHkdYiuk	
0-01	101	x_1	x_2
		0-00	1-00
		0-52	0-48
		1-00	0-00
0-02	51	x_1	x_2
		0-52	0-48
		1-00	0-00
		0-00	1-00
0-03	34	x_1	x_2
		0-52	0-48
		1-00	0-00
		0-00	1-00
0-04	26	x_1	x_2
		0-52	0-48
		1-00	0-00
		0-00	1-00
0-05	21	x_1	x_2
		0-50	0-50
		1-00	0-00
		0-00	1-00

x_1 vkj x_2 bVjD'ku Ve d lkFk tgk] θ_1] θ_2 vkj θ_3 feJ.k l tM rhu ijkehVj gA bl çdkj] bl fLFkfr e çkIr lrlr Mh&v,lVhey vfHkdYiuk rhu ju e ,d vfHkdYiuk gA çkjfHkd ijkehVj vuekuk d lkFk fofHkUu oru of) d fy, ,YxkfjFke d ek/;e l mRiUu vfHkdYiuk fuEu rkfydk 1 e lphc) gA

mijkä n'kkb xb rkfydk e] ;g fn[kk;k x;k g fd ,YxkfjFke dk fofHkUu bUØheV of) d fy, pyk;k x;k g] tl kfd 0-01] 0-02] 0-03] 0-04 vkj Mh&v,lVhey vfHkdYiuk leku jgrk gA Øe'k% ckbujh CyM vkj 'k) feJ.k l;ä bUØheV of) 0-05 d fy, vfHkdYiuk fcn FkkMk cny tkrk gA blh çdkj fodfLr ,YxkfjFke dk mi;kx djd nlj e,My d fy, Hkh vfHkdYiuk; fuek.k dj ldr gA

fu"d"k

x.kkRed çfrfØ;k ifjoru'khy fLFkfr d ekey e] pj vkerkj ij lkekU; forj.k dk ikyu ugh djr gA ;gk] bl v/;u e y,ftfLVd e,My dk mi;kx fd;k x;k g vkj y,ftfLVd e,My dk mi;kx djd feJ.k ç;kxk d fy, vfHkdYiuk; çkIr fd, x, gA fodfLr Mh&v,lVhey vfHkdYiukvk dh lpuk nh xb gAmijkä fLFkfr e] çLrkfor l'kkf/kr QMkjko ,YxkfjFke ly'ku LiIl d :i e dfMMV lV ij cgrj dke djrk gA ;g dfMMV lV feJ.k ç;kxk d fy, fujrj vfHkdYiuk LFkku dk çfrfuf/kRo djrk gA lHkh lx.kuk, R l,¶Vo;j dk mi;kx djd dh xb gA

lUnHk

Atkinson, A.C. and Donev, A.N. (1989). The construction of exact d-optimum experimental designs with application to blocking response surface designs. Biometrika. 76(3): 515-526.

Cornell, J.A. (2002). Experiments with Mixtures. Third Edition. Wiley, New York.

Scheffé, H. (1958). Experiments with mixtures. Journal of Royal Statistical Society. Series B. 20: 344-360.

Scheffé, H. (1963). Simplex-centroid designs for experiments with mixtures. Journal of Royal Statistical Society. Series B. 20: 344-360.