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nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiukvk d fuek.k

ugkrkb okeujko vxk'k[1][2] fluh oxhl[1] fouk;dk[1][2] ekgEen gk#u[1]* noUæ dekj[1]

lkjk”k

i"VHkfe% nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M ¼DNPBIB½ vfHkdYiuk dk [k.M dh rhu ç.kkfy;k] ft le v mipkjk e l çR;d dk r ckj nkgjk, tku dh 0;oLFkk d :i e ifjHkkf"kr fd;k x;k g] ;fn igyh ç.kkyh d çR;d [k.M e nljh ç.kkyh d m_1 [k.M gkr g vkj nljh ç.kkyh d çR;d [k.M e] rhljh ç.kkyh d m_2 [k.M lfEefyr gkr gA DNPBIB vfHkdYiuk dk çR;d pj.k] Lor= :i l 0;ogkj dh xb] ,d vkf'kd :i l lrfyr vi.k [k.M ¼PBIB½ vfHkdYiuk gA

fof/k;k% rhu ?kVd vfHkdYiuk vFkkr ¼i½ [k.Mk dh igyh vkj nljh ç.kkyh dk vun[kk djr g,] çR;d vkdkj k_3 ¼< v½ bdkb;k d b_3 [k.Mk d lkFk ,d PBIB vfHkdYiuk] λ_{3i} mipkjk dh fdlh Hkh tksM+h dh lgefr tk çR;d d i^{th} lg;kxh g] ¼ii½ [k.Mk dh igyh vkj rhljh ç.kkyh dk vun[kk djr g,] çR;d vkdkj k_2 ¼< v½ bdkb;k d b_2 [k.Mk d lkFk ,d vkj PBIB vfHkdYiuk ft le λ_{3i} nk mipkjk dh lgefr g tk ,d nlj d lg;kxh g ,o ¼iii½ nlj vkj rhlj [k.Mk dh ç.kkyh dk vun[kk djr g,] çR;d vkdkj dh k_1 ¼< v½ bdkb;k d b_1 [k.M d lkFk ,d rhljh PBIB vfHkdYiuk] ,d nlj d igy lg;kxh d Hkhrj nk mipkjk dh λ_{1i} lgefr çklr dh tkrh g A

ifj.kke% ;g v/;;u DNPBIB vfHkdYiuk d fuek.k dh nk ub lkeU; fof/k;k çnku djrk g] tgk ?kVd vfHkdYiuk nk ;k rhu lg;kxh oxk d lkFk PBIB vfHkdYiuk gkrh gA ,l vfHkdYiuk fodflr dju d fy, leg foHkkT; vkj vk;rkdkj l?k ;ktukvk dk mi;kx fd;k x;k gA fof/k;k dk mi;ä mnkgj.kk d lkFk n'kk;k x;k gA

“kCn dth% n{krk] leg foHkkT; l?k ;ktuk] vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk] vk;rkdkj l?k ;ktukA

On Construction of Doubly Nested Partially Balanced Incomplete Block Designs

Nehatai Wamanrao Agashe[1,2], Cini Varghese[1], Vinayka[1,2], Mohd Harun[1], Devendra Kumar[1] **10.18805/BKAP702**

ABSTRACT

Background: A doubly nested partially balanced incomplete block (DNPBIB) design is defined as an arrangement of v treatments each replicated r times in three systems of blocks if, each block of the first system contains m_1 blocks of second system and each block of the second system contains m_2 blocks of the third system. Each stage of DNPBIB design, treated independently, is a PBIB design.

Methods: Three component designs *viz*., (i) ignoring the first and second system of blocks, a PBIB design with b_3 blocks each of size k_3 (< v) units with, λ_{3i} concurrences of any pair of treatments which are i^{th} associates of each other, (ii) ignoring first and third system of blocks, another PBIB design with b_2 blocks each of size k_2(< v) units with λ_{3i} concurrence of two treatments which are i^{th} associates of each other and (iii) ignoring the second and third system of blocks, a third PBIB design with b_1 block each of size k_1 (< v) units with, λ_{1i} concurrences of two treatments within first associate of each other, are obtained.

Result: This study provides two new general methods of constructing DNPBIB designs, where the component designs are PBIB designs with two or three associate classes. Group divisible and rectangular association schemes have been used for developing such designs. Methods are illustrated with appropriate examples.

Key words: Efficiency, Group divisible association scheme, Partially balanced incomplete block design, Rectangular association scheme.

ifjp;

oKkfud ijh{k.kk dh lkf[;dh; vfHkdYiuk e/;ku e j[kh tku okyh lcl egRoi.k leL;k ijh{k.kkRed lkefx;k e fofo/krk g A ;fn fdlh ijh{k.k dh ;ktuk cukr le; bl ij Bhd l fopkj ugh fd;k tk,] rk bl okLrfod mipkj vrj vLi"V gku dh lHkkouk g] tc rd fd mipkjk dh l[;k i;kIr u gk] mudk irk yxkuk dfBu gkxkA dHkh&dHkh] ,d miæo dkjd mRiUu gkrk g] tk ijh{k.kdrk d fy,

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v#fpdj gkr g, Hkh ijh{k.kkRed lkexh e ifjoru'khyrk e egRoi.k ;kxnku nrk gA [k.Mh;dj.k ,d ,lh rduhd g ftldk mi;kx fdlh [k.M d Hkhrj ijh{k.kkRed bdkb;k dh ,d:irk çkIr dju d fy, fd;k tkrk g rkfd vr[k.Mh; tkudkjh dk mi;kx djd mipkj fo"kerkvk dk vf/kd d'kyrk l vueku yxk;k tk ldA lcl ljy vkj lcl vf/kd bLreky dh tku okyh [k.M vfHkdYiuk ,d ;k–fPNd i.k [k.M ¼RCB½ vfHkdYiuk gA ;k–fPNd i.k [k.M vfHkdYiuk lcl n{k vfHkdYiuk g D;kfd mipkj fo"kerk d lkFk&lkFk [k.M fo"kerk dk vueku yxku e tkudkjh dh dkb gkfu ugh gkrh gA ;|fi] tc fdlh ijh{k.k e mipkjk dh l[;k c<rh g] rk ;k–fPNd i.k [k.M vfHkdYiuk d [k.M cM gk tkr g; vkj [k.Mk d Hkhrj ,d:irk cuk, j[kuk lHko ugh gk ikrk gA ,lh fLFkfr;k e vi.k [k.M vfHkdYiukvk dk ykHkçn :i l mi;kx fd;k tk ldrk g D;kfd mud [k.M NkV gkr gA vi.k [k.M vfHkdYiukvk dh J.kh e] ;Vl ¼1936½ }kjk nh xb lrfyr vi.k [k.M vfHkdYiuk ¼BIB½ lcl ljy gA yfdu çR;d çkpyh; l;ktu d fy, lrfyr vi.k [k.M vfHkdYiuk miyC/k ugh gA bld vfrfjä] Hky gh fdlh fn, x, mipkjk dh l[;k ¼v½ vkj [k.M vkdkj ¼k½ d fy, lrfyr vi.k [k.M vfHkdYiuk mifLFkr gk] bld fy, cgr vf/kd çfr–fr dh vko';drk gk ldrh gA bl leL;k dk nj dju d fy, ckl vkj uk;j ¼1939½ u f}pjh;] le&çfr–fr ,o mfpr vfHkdYiukvk dh ,d J.kh çLrr dh] ftUg vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk, ¼PBIBD½ dgk tkrk gA

uhfMr [k.M vfHkdYiuk o [k.M vfHkdYiuk gkrh g ftue [k.M dh nk ç.kkfy;k gkrh g ftue [k.M dh nljh ç.kkyh igyh ç.kkyh d Hkhrj uhfMr gkrh gA uhfMr lrfyr vi.k [k.M vfHkdYiuk ¼çhl] 1967½ vkj uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk ¼gkey ,oj,fcUlu] 1975½ uhfMr [k.M vfHkdYiuk d nk egRoi.k ox cukrh gA ,lh fLFkfr mRiUu gk ldrh g tc ijh{k.k lkexh d mi&[k.M Lrj e fHkUurk dk rhljk lkr mifLFkr gk] ml fLFkfr e nkgjh uhfMr [k.M vfHkdYiuk dk mi;kx fd;k tkrk gA nkgjh uhfMr [k.M vfHkdYiuk ,d [k.M vfHkdYiuk g ftle [k.Mh;dj.k dh rhu ç.kkfy;k lfEefyr gkrh g] ftle [k.Mh;dj.k dh igyh ç.kkyh dk nljh ç.kkyh d Hkhrj uhfMr fd;k tkrk g vkj [k.Mh;dj.k dh nljh ç.kkyh dk [k.Mh;dj.k dh rhljh ç.kkyh d Hkhrj uhfMr fd;k tkrk gA bl vfHkdYiuk dk mi;kx fdlh ijh{k.k dh ijh{k.kkRed lkexh e mifLFkr fHkUurk d rhu lkrk dk lekIr dju d fy, fd;k tkrk gA fuEufyf[kr dN ijh{k.kkRed fLFkfr;k gA

mnkgj.k 1& dVkb mi& [k.M d vulkj dh tkrh g vkj çR;d mi&[k.M l ,df=r çfrn'kk dk fofHkUu rduhf'k;uk }kjk ç;kx'kkyk e mudh lkexh d fy, fo'y"k.k fd;k tkuk gA rduhf'k;uk d dkj.k fHkUurk dk fu;f=r dju d fy,] bl ,d vU; vojk/kd dkjd d :i efy;k tk ldrk gA blfy, ,d mi&[k.M d Hkhrj bdkb;k dk LFkkfir dju dh vko';drk gk ldrh g ¼eMy bR;kfn] 2012½A

mnkgj.k 2& ,d vfHk;kf=dh ijh{k.k e] fofHkUu ihlu okyh e'khuk dh n{krk dk v/;;u dju d fy, ijh{k.kkRed lkexh dk vyx&vyx okrkoj.kk d vulkj lefgr fd;k tk ldrk g] vyx&vyx okrkoj.kk dk vyx&vyx cpk d vulkj lefgr fd;k tk ldrk g vkj vkx vyx&vyx cpk dk vyx&vyx v,ijVjk d vulkj legh–r fd;k tk ldrk gA ,lh ijh{k.kkRed fLFkfr;k l fuiVu d fy, nkgjh uhfMr [k.M vfHkdYiukvk dk mi;kx fd;k tkrk g ¼elu bR;kfn] 2003½ tgk fofo/krkvk d rhu lkr gkr g] fHkUurk d rhlj lkr ¼mi&mi [k.M½ dk nlj ¼mi& [k.M½ d Hkhrj vkj nlj dk igy ¼[k.M½ d Hkhrj uhfMr fd;k tkrk gA

çhl ¼1967½ u uhfMr lrfyr vi.k [k.M vfHkdYiuk çLrr dh vkj uhfMr lrfyr vi.k [k.M vfHkdYiukvk dk ijh{k.k vkj =fV lek/kku fn;kA gkey ,oj,fcUlu ¼1975½ u uhfMr lrfyr vi.k [k.M vfHkdYiukvk dk ifjHkkf"kr fd;k vkj bu vfHkdYiukvk d fuek.k d rjhd fn,A ftEck ,o dfjdh ¼1983½] M bR;kfn ¼1986½ u uhfMr lrfyr vi.k [k.M vfHkdYiukvk d fuek.k d fy, lkekU; rjhdk dk ,d lV fodflr fd;kA cuth ,o dkx;kek ¼1993½ vkj lkgk bR;kfn ¼1998½ u uhfMr lrfyr vkj vkf'kd :i l lrfyr vi.k [k.M vfHkdYiukvk d fuek.k d dN 0;ofLFkr rjhd çLrr fd,A xlrk ¼1993½ u n[kk fd uhfMr [k.M vfHkdYiuk d b"Vre ifj.kke doy mi&[k.M vfHkdYiuk d b"Vre ifj.kke ij fuHkj FkA dkx;kek bR;kfn ¼1995½ u nkgjhdj.k dh rduhd dk mi;kx djd uhfMr lrfyr vi.k [k.M vfHkdYiuk vkj 2& lg;kxh ox uhfMr lrfyr vi.k [k.M vfHkdYiuk çkIr dju dh dN çfØ;k, nhA fQfyi bR;kfn ¼1997½ u çfrcf/kr nkgjhdj.k dh rduhd dk mi;kx djd nk lg;kxh uhfMr lrfyr vi.k [k.M vfHkdYiukvk dh vo/kkj.kk dk uhfMr Vh&

lg;kxh vkf'kd lrfyr vi.k [k.M vfHkdYiukvk rd foLrkfjr fd;k A lrifr ½2001½ ,o e,xu bR;kfn ½2001½ u uhfMr lrfyr vi.k [k.M vfHkdYiukvk dh ,d foLrr leh{kk çLrr dh vkj bu vfHkdYiukvk dh lph çnku dhA lrifr ,o çlkn ½2004½ vkj çlkn ½2019½ u 2& vkj 3≶kxh ox uhfMr lrfyr vi.k [k.M vfHkdYiukvk d fuek.k d dN u, rjhd çkIr fd, vkj uhfMr lrfyr vi.k [k.M vfHkdYiukvk dh lph çnku dhA nkgjh uhfMr lrfyr vi.k [k.M dh vo/kkj.kk çhl bR;kfn ½1999½ }kjk çLrr dh xb FkhA eMy bR;kfn ½2012½ u nkgjh uhfMr lrfyr vi.k [k.M vfHkdYiuk çLrr dh vkj nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk d fuek.k d dN lkekU; rjhd fn,A

çLrr v/;;u e] nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk d fuek.k dh nk fof/k;k fodflr dh xb gA bu fof/k;k dk fuEufyf[kr vuHkkx e le>k;k x;k gA

e,My ,o ijh{k.kkRed foU;kl ifjHkk"kk

nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk dk [k.M dh rhu ç.kkfy;k] ftle v mipkjk e l çR;d dk r ckj nkgjk, tku dh 0;oLFkk d :i e ifjHkkf"kr fd;k tk ldrk g] ;fn

1- igyh ç.kkyh d çR;d [k.M e nljh ç.kkyh d m_1 [k.M gkr g vkj nljh ç.kkyh d çR;d [k.M e rhljh ç.kkyh d m_2 [k.M gkr g]

2- ;fn ge [k.Mk dh igyh vkj nljh ç.kkyh dk vun[kk djr g] rk ;g k bdkb;k d vkdkj d b_3 [k.Mk dh ,d vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk cukrh g ftle mipkj d fdlh Hkh tkM dh λ_{3i} lgefr gkrh g tk ,d nlj d io lg;kxh gkr gA

3- çhl bR;kfn ½2012½ d vulkj [k.Mk dh igyh vkj rhljh ç.kkyh dk vun[kk djr g,] ;g nk mipkjk dh λ_{2i} lgefr d lkFk çR;d vkdkj dh k_2 bdkb;k oky b_2 [k.Mk d lkFk ,d vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk cukrh g tk ,d nlj d io lg;kxh g vkj [k.Mk dh nljh vkj rhljh ç.kkyh dk vun[kk djr g,] ,d nlj d igy lg;kxh d Hkhrj nk mipkjk dh λ_{1i} lgefr d lkFk b_1 [k.Mk] çR;d d k_1 bdkb;k d vkdkj dh ;g ,d vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk curh gA

nkgjh uhfMr lrfyr [k.M vfHkdYiuk e,My

fuEufyf[kr fuf'pr çHkko e,My ij fopkj fd;k tk ldrk g%

$$y_{ij"(j'(j))u} = \mu+\iota_i+\beta_j+\eta_{j'(j)}+\gamma_{j"(j'j))}+\varepsilon_{j"(j'j))u} \quad \ldots\ldots\ldots\ldots(1)$$

tgk $y_{ij"(j'(j))u}$ io ¼i=1,2,...,v½ mipkj dk çkIr djr g, jo ¼i=1,2,...,v½ [k.M d Hkhrj j'o ¼j'=1,2,...,q_j½ mi&[k.M e j"o ¼j"=1,2,...,$s_{j'(j)}$½ mi& mi&[k.M d vnj uhfMr u oh bdkb ij voykdu dk n'kkrk g] μ lkekU; ek/; g] ι_i iok mipkj çHkko g] β_j jo [k.M dk çHkko g] $\eta_{j'(j)}$ jo [k.M d Hkhrj j'o mi&[k.M dk çHkko g] $\gamma_{j"(j'j))}$] jo [k.M d Hkhrj uhfMr j'o mi&[k.M d vnj uhfMr j" o mi& mi&[k.M dk çHkko g] $\varepsilon_{j"(j'j))u}$ =fV ing tk Lor= vkj leku :i l 'kU; ek/; vkj fLFkj σ^2 fopj.k d lkFk lkekU; forj.k dk ikyu djr g, forfjr gA

nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiukvk gr ljpuk i)fr;k

bl vuHkkx egeu nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiukvk d fuek.k d fy, ,d fof/k fodflr dh gA ;g fof/k leg foHkkT; lgp; ¼GD association½ ;ktuk ij vk/kkfjr gA

çFke i)fr% eku yhft, v=mn; m ≥ 2, n=4] nk&ox leg foHkkT; lgp; ;ktuk ij ifjHkkf"kr mipkj tk fuEukulkj çnf'kr gkr g %

1	m +1	2m+1	3m+1
2	m+2	2m+2	3m+2
-	-	-	-
m	2m	3m	4m

vc] leg foHkkT; lgp; ;ktuk d vk/kkj ij ,d nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk dk fuek.k fuEufyf[kr çfØ;k dk mi;kx djd fd;k tk ldrk gA

pj.k 1& ,d gh ifä d lHkh mipkj ,d nlj d igy lg;kxh g vkj ble 4¼m&1½ vkdkj d leku nlj lg;kxh lfEefyr g ftudk mi;kx lHkh lHkkfor l;ktuk dk cuku d fy, fd;k tkrk gA

pj.k 2& ; l;ktu ;g lfuf'pr djr g fd çR;d mipkj doy viu nlj lg;kfx;k d lkFk fcuk fdlh tkMh d nkgjko d vkrk gA

pj.k 3& ;fn pkj mipkj] eku yhft,] ¼$\alpha, \beta, \gamma, \delta$½ ,d nlj d igy lg;kxh g] rk mud nlj lg;kxh] ¼$\theta, \psi, \eta,$

ω½ ekuk] tk viu Hkhrj Hkh igy lg;kxh gA

pj.k 4& l;ktu ¼α,θ½] ¼β,ψ½] ¼γ,η½ and ¼δ,ω½ dk mi&mi&[k.M d :i e y] ftue l çR;d dk vkdkj nk g] nk& nk mi&[k.Mk dk foy; dj] çR;d dk pkj vkdkjk [(α,θ),¼β,ψ½] ,o [(γ,η), (δ,ω)] e mi&[k.M cuk fn;k x;k g] fQj mUg vkdkj vkB dk ,d lij [k.M {[(α, θ), (β,ψ)] [(γ,η), (δ,ω)]} e y tk;k x;k blh çdkj] vU; lHkkfor lij [k.M {[(α,ψ), (β,θ)][(γ,ω), (δ,η)]} ,o {[(α,ω), (δ,θ)] [(β,η), (γ,ψ)]} gA

pj.k 5& lHkh v ¾ 4m mipkjk d fy, pj.k 1 l 4 dk ikyu djr g,] v ¾ 4m] r ¾ 12¼m&1½] b_1 ¾ 12¼m_2½] b_2¾ 24¼m_2½] b_3¾ 48¼m_2½] k_1¾ 8] k_2¾ 4] k_3¾ 2] λ_{11}¾ 12¼m-1½] λ_{12}¾4 ¼m&1½] λ_{21}¾ 4¼m&1½] λ_{22}¾6] λ_{31}¾0] λ_{32}¾3 çkpyk d lkFk [kMh;dj.k d çR;d Lrj ij leg foHkkT; lgp; ;ktuk d vk/kkj ij ,d nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk çkIr gkrh gA

mnkgj.k& eku yhft, fd v ¾ 12 mipkjk d fy, m ¾ 3 vkj n ¾ 4 leg foHkkT; lgp; ;ktuk ij ifjHkkf'kr g] vFkkr]

1	2	3	4
5	6	7	8
9	10	11	12

vc i)fr dh çfØ;k d vulkj] ge leg foHkkT; lgp; ;ktuk d vk/kkj ij nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk feyrh g] ftle vfHkdYiuk d çkpy] v –12, r–24, β_1 – 36, β_2 – 72, β_3 – 144, k_1–8, k_2 – 4, k_3– 2, λ_{11}–24, λ_{12}–12, λ_{21}–8, λ_{22}–6, λ_{31}– 0, λ_{32}– 3 gA vfHkdYiuk dk foU;kl bl çdkj gA

¿¾1]	5½]	¼2]	6½],	[¾3]	7½]	¼4	8½]À
¿¾1]	9½]	¼2]	10½],	[¾3]	11½]	¼4	12½]À
¿¾5]	9½]	¼6]	10½],	[¼7]	11½]	¼8	12½]À
¿¾1]	6½]	¼2]	5½],	[¾3]	8½]	¼4	7½]À
¿¾1]	10½]	¼2]	9½],	[¾3]	12½]	¼4	11½]À
¿¾5]	10½]	¼6]	9½],	[¼7]	12½]	¼8	11½]À
¿¾1]	7½]	¼3]	5½],	[¼2]	8½]	¼4	6½]À
¿¾1]	11½]	¼3]	9½],	[¼2]	12½]	¼4	10½]À
¿¾5]	11½]	¼7]	9½],	[¼6]	12½]	¼8	10½]À
¿¾1]	8½]	¼4]	5½],	[¼2]	7½]	¼3	6½]À
¿¾1]	12½]	¼4]	9½],	[¼2]	11½]	¼3	10½]À
¿¾5]	12½]	¼8]	9½],	[¼6]	11½]	¼7	10½]À
¿¾1]	5½]	¼3]	7½],	[¼2]	6½]	¼4	8½]À
¿¾1]	9½]	¼3]	11½],	[¼2]	10½]	¼4	12½]À
¿¾5]	9½]	¼7]	11½],	[¼6]	10½]	¼8	12½]À
¿¾1]	6½]	¼3]	8½],	[¼2]	5½]	¼4	7½]À
¿¾1]	10½]	¼3]	12½],	[¼2]	9½]	¼4	11½]À
¿¾5]	10½]	¼7]	12½],	[¼6]	9½]	¼8	11½]À
¿¾1]	7½]	¼2]	8½],	[¼3]	5½]	¼4	6½]À
¿¾1]	11½]	¼2]	12½],	[¼3]	9½]	¼4	10½]À
¿¾5]	11½]	¼6]	12½],	[¼7]	9½]	¼8	10½]À
¿¾1]	8½]	¼2]	7½],	[¼4]	5½]	¼3	6½]À
¿¾1]	12½]	¼2]	11½],	[¼4]	9½]	¼3	10½]À
¿¾5]	12½]	¼6]	11½],	[¼8]	9½]	¼7	10½]À
¿¾1]	5½]	¼4]	8½],	[¼2]	6½]	¼3	7½]À
¿¾1]	9½]	¼4]	12½],	[¼2]	10½]	¼3	11½]À
¿¾5]	9½]	¼8]	12½],	[¼6]	10½]	¼7	11½]À
¿¾1]	6½]	¼4]	7½],	[¼2]	5½]	¼3	8½]À
¿¾1]	10½]	¼4]	11½],	[¼2]	9½]	¼3	12½]À
¿¾5]	10½]	¼8]	11½],	[¼6]	9½]	¼7	12½]À
¿¾1]	7½]	¼4]	6½],	[¼3]	5½]	¼2	8½]À
¿¾1]	11½]	¼4]	10½],	[¼3]	9½]	¼2	12½]À
¿¾5]	11½]	¼8]	10½],	[¼7]	9½]	¼6	12½]À
¿¾1]	8½]	¼3]	6½],	[¼4]	5½]	¼2	7½]À
¿¾1]	12½]	¼3]	10½],	[(4]	9½]	¼2	11½]À
¿¾5]	12½]	¼7]	10½],	[¼8]	9½]	¼6	11½]À

f}rh; i)fr eku yhft, v ¾ mn mipkj vk;rkdkj lgp; ;ktuk ij ifjHkkf"kr fd, x, g A lgp; ;ktuk d lkFk m × n lj.kh ij v ¾ mn¼n ≥ 4] m¾ n + 1½ mipkj dh 0;oLFkk dh xbA

fuEufyf[kr pj.k nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk nr g A

1. çR;d ifä Ri, Ri, i, ¾ ¼1½m dk n vkdkj d ,d mi&[k.M d :i e yA

2. puh xb ifä ¼Ri½ mipkjk dk vun[kk dj LrHk mipkjk dk mi;kx djd n vkdkj d mi&mi [k.M fy[k vkj mUg C1¼i½] C2¼i½...] Cn¼i½ uke n A

3. bu n mi&mi [k.Mk ij

¿[Ri] C_{1i}]] [C_{2i}] C_{3i}]À

¿[Ri] C_{1i}]] [C_{2i}] C_{4i}]À

çdkj dh fuf'pr ifä Ri d lkFk lrfyr vi.k [k.M vfHkdYiuk ¼v' ¾ n] b'] r',k' ¾ 3] λ'½ ij fopkj dj tkrk^ [k.M nrk gA

4. fuf'pr ifä Ri ftle çR;d 4n vkdkj d k'b' ¾ 3b' [k.M lfEefyr g] d fy, k' ¾ 3 ckj vrjifjoruh; LrHk yA

;g çfØ;k lHkh m ifä;k d fy, vk;rkdkj lgp; ;ktuk d vk/kkj ij] v= mn] v ≥ 4] r= 4nr'] m= n + 1, n= v', b_1=3mb', b_2=6mb', b_3= 12mb', k_1= 4n, k_2= 2n, k_3= n, λ_{11}= 3(nλ'- b'), λ_{12}= mr', λ_{13}=12r', λ_{21}=3b' + nλ', λ_{22}= r' (3n-1), λ_{23}= 4r, λ_{31}=3b', λ_{32}= 3(n-1)r', λ33= 0 çkpyk d lkFk nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk cuku d fy, ykx dh tkrh gA

mnkgj.k& eku yhft, fd v¾ 20 mipkjk d fy, m¾ 4 vkj n¾ 5] vk;rkdkj lgp; ;ktuk ij ifjHkkf"kr g] vFkkr]

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

vc i)fr 2 dh çfØ;k d vulkj] ge v ¾ 20] n = 4] r¾ 48] m¾ 5] n¾ 4] b_1¾ 60] b_2¾ 120] b_3¾ 240] k_1¾ 16] k_2¾ 8] k_3¾ 4] λ_{11}¾ 36] λ_{12}¾ 45] λ_{13}¾ 36] λ_{21}¾ 20] λ_{22} ¾ 33] λ_{23}¾ 20] λ_{31}¾ 12] λ_{32}¾ 36] λ_{33}¾ 0 çkpyk d lkFk vk;rkdkj lgp; ;ktuk d vk/kkj ij nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiuk feyrh gA

{[(1,	6,	11,	16),	(2,	3,	4,	5)],	[(7,	8,	9,	10),	(12,	13,	14,	15)]}
{[(1,	6,	11,	16),	(2,	3,	4,	5)],	[(7,	8,	9,	10),	(17,	18,	19,	20)]}
{[(1,	6,	11,	16),	(2,	3,	4,	5)],	[(12,	13,	14,	15),	(17,	18,	19,	20)]}
{[(1,	6,	11,	16),	(7,	8,	9,	10)],	[(12,	13,	14,	15),	(17,	18,	19,	20)]}
{[(1,	6,	11,	16),	(7,	8,	9,	10)],	[(2,	3,	4,	5),	(12,	13,	14,	15)]}
{[(1,	6,	11,	16),	(7,	8,	9,	10)],	[(2,	3,	4,	5),	(17,	18,	19,	20)]}
{[(1,	6,	11,	16),	(12,	13,	14,	15)],	[(2,	3,	4,	5),	(17,	18,	19,	20)]}
{[(1,	6,	11,	16),	(12,	13,	14,	15)],	[(7,	8,	9,	10),	(17,	18,	19,	20)]}
{[(1,	6,	11,	16),	(12,	13,	14,	15)],	[(2,	3,	4,	5),	(7,	8,	9,	10)]}
{[(1,	6,	11,	16),	(17,	18,	19,	20)],	[(2,	3,	4,	5),	(7,	8,	9,	10)]}
{[(1,	6,	11,	16),	(17,	18,	19,	20)],	[(2,	3,	4,	5),	(12,	13,	14,	15)]}
{[(1,	6,	11,	16),	(17,	18,	19,	20)],	[(7,	8,	9,	10),	(12,	13,	14,	15)]}
{[(2,	7,	12,	17),	(1,	6,	11,	16)],	[(3,	8,	13,	18),	(4,	9,	14,	19)]}
{[(2,	7,	12,	17),	(1,	6,	11,	16)],	[(3,	8,	13,	18),	(5,	10,	15,	20)]}
{[(2,	7,	12,	17),	(1,	6,	11,	16)],	[(4,	9,	14,	19),	(5,	10,	15,	20)]}
{[(2,	7,	12,	17),	(3,	8,	13,	18)],	[(4,	9,	14,	19),	(5,	10,	15,	20)]}
{[(2,	7,	12,	17),	(3,	8,	13,	18)],	[(1,	6,	11,	16),	(4,	9,	14,	19)]}
{[(2,	7,	12,	17),	(3,	8,	13,	18)],	[(1,	6,	11,	16),	(5,	10,	15,	20)]}
{[(2,	7,	12,	17),	(4,	9,	14,	19)],	[(1,	6,	11,	16),	(5,	10,	15,	20)]}
{[(2,	7,	12,	17),	(4,	9,	14,	19)],	[(3,	8,	13,	18),	(5,	10,	15,	20)]}
{[(2,	7,	12,	17),	(4,	9,	14,	19)],	[(3,	8,	13,	18),	(1,	6,	11,	16)]}
{[(2,	7,	12,	17),	(5,	10,	15,	20)],	[(3,	8,	13,	18),	(1,	6,	11,	16)]}
{[(2,	7,	12,	17),	(5,	10,	15,	20)],	[(4,	9,	14,	19),	(1,	6,	11,	16)]}
{[(2,	7,	12,	17),	(5,	10,	15,	20)],	[(4,	9,	14,	19),	(3,	8,	13,	18)]}
{[(3,	8,	13,	18),	(1,	2,	4,	5)],	[(6,	7,	9,	10),	(11,	12,	14,	15)]}

Table Continue...

Table Continue...

{[(3,	8,	13,	18),	(1,	2,	4,	5)],	[(6,	7,	9,	10),	(16,	17,	19,	20)]}
{[(3,	8,	13,	18),	(1,	2,	4,	5)],	[(11,	12,	14,	15),	(16,	17,	19,	20)]}
{[(3,	8,	13,	18),	(6,	7,	9,	10)],	[(11,	12,	14,	15),	(16,	17,	19,	20)]}
{[(3,	8,	13,	18),	(6,	7,	9,	10)],	[(1,	2,	4,	5),	(11,	12,	14,	15)]}
{[(3,	8,	13,	18),	(6,	7,	9,	10)],	[(1,	2,	4,	5),	(16,	17,	19,	20)]}
{[(3,	8,	13,	18),	(11,	12,	14,	15)],	[(1,	2,	4,	5),	(16,	17,	19,	20)]}
{[(3,	8,	13,	18),	(11,	12,	14,	15)],	[(6,	7,	9,	10),	(16,	17,	19,	20)]}
{[(3,	8,	13,	18),	(11,	12,	14,	15)],	[(1,	2,	4,	5),	(6,	7,	9,	10)]}
{[(3,	8,	13,	18),	(16,	17,	19,	20)],	[(1,	2,	4,	5),	(6,	7,	9,	10)]}
{[(3,	8,	13,	18),	(16,	17,	19,	20)],	[(1,	2,	4,	5),	(11,	12,	14-	15)]}
{[(3,	8,	13,	18),	(16,	17,	19,	20)],	[(6,	7,	9,	10),	(11,	12,	14,	15)]}
{[(4,	9,	14,	19),	(1,	2,	3,	5)],	[(6,	7,	8,	10),	(11,	12,	13,	15)]}
{[(4,	9,	14,	19),	(1,	2,	3,	5)],	[(6,	7,	8,	10),	(16,	17,	18,	20)]}
{[(4,	9,	14,	19),	(1,	2,	3,	5)],	[(11,	12,	13,	15),	(16,	17,	18,	20)]}
{[(4,	9,	14,	19),	(6,	7,	8,	10)],	[(11,	12,	13,	15),	(16,	17,	18,	20)]}
{[(4,	9,	14,	19),	(6,	7,	8,	10)],	[(1,	2,	3,	5),	(11,	12,	13,	15)]}
{[(4,	9,	14,	19),	(6,	7,	8,	10)],	[(1,	2,	3,	5),	(16,	17,	18,	20)]}
{[(4,	9,	14,	19),	(11,	12,	13,	15)],	[(1,	2,	3,	5),	(16,	17,	18,	20)]}
{[(4,	9,	14,	19),	(11,	12,	13,	15)],	[(6,	7,	8,	10),	(16,	17,	18,	20)]}
{[(4,	9,	14,	19),	(11,	12,	13,	15)],	[(1,	2,	3,	5),	(6,	7,	8,	10)]}
{[(4,	9,	14,	19),	(16,	17,	18,	20)],	[(1,	2,	3,	5),	(6,	7,	8,	10)]}
{[(4,	9,	14,	19),	(16,	17,	18,	20)],	[(1,	2,	3,	5),	(11,	12,	13,	15)]}
{[(4,	9,	14,	19),	(16,	17,	18,	20)],	[(6,	7,	8,	10),	(11,	12,	13,	15)]}
{[(5,	10,	15,	20),	(1,	2,	3,	4)],	[(6,	7,	8,	9),	(11,	12,	13,	14)]}
{[(5,	10,	15,	20),	(1,	2,	3,	4)],	[(6,	7,	8,	9),	(16,	17,	18,	19)]}
{[(5,	10,	15,	20),	(1,	2,	3,	4)],	[(11,	12,	13,	14),	(16,	17,	18,	19)]}
{[(5,	10,	15,	20),	(6,	7,	8,	9)],	[(11,	12,	13,	14),	(16,	17,	18,	19)]}
{[(5,	10,	15,	20),	(6,	7,	8,	9)],	[(1,	2,	3,	4),	(11,	12,	13,	14)]}
{[(5,	10,	15,	20),	(6,	7,	8,	9)],	[(1,	2,	3,	4),	(16,	17,	18,	19)]}
{[(5,	10,	15,	20),	(11,	12,	13,	14)],	[(1,	2,	3,	4),	(16,	17,	18,	19)]}
{[(5,	10,	15,	20),	(11,	12,	13,	14)],	[(6,	7,	8	9),	(16,	17,	18,	19)]}
{[(5,	10,	15,	20),	(11,	12,	13,	14)],	[(1,	2,	3,	4),	(6,	7,	8,	9)]}
{[(5,	10,	15,	20),	(16,	17,	18,	19)],	[(1,	2,	3,	4),	(6,	7,	8,	9)]}
{[(5,	10,	15,	20),	(16,	17,	18,	19)],	[(1,	2,	3,	4),	(11,	12,	13,	14)]}
{[(5,	10,	15,	20),	(16,	17,	18,	19)],	[(6,	7,	8,	9),	(11,	12,	13,	14)]}

ifjpp k

leg foHkkT; vkj vk;rkdkj lgp; ;k tukvk dk mi;kx djd nkgjh uhfMr vkf'kd :i l lrfyr vi.k [k.M vfHkdYiukvk d fuek.k d fy, nk i)fr;k çLrkfor dh xb gA bu vfHkdYiukvk dk fuek.k djuk vklku g vkj ; çkpfyd l;k tuk dh ,d foLrr J[kyk dk lfEefyr djrh gA tc nk vfrfjä miæo dkjd ijh{k.kkRed vfHkdYiuk d [k.M d Hkhrj inkuØfer :i l fufgr gkr g rc bUg ijh{k.kk e llk/ku&cpr d fodYi d :i e mi;kx fd;k tk ldrk gA

lnHk

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