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,dkfèkd fu;=.kk d lkFk ijh{k.k mipkjk dh ryuk dju d fy, vi.kZ Cy,d fMtkbu

fouk;dk[1,2] ch-,u- eMy[3] jktæ çlkn[4] ,y-,u- fou;dekj[1] ih- ejyh[2] vej"k[2] "ork dekjh[2]

lkjk"k

i'BHkfe% ;g vky[k ,d l vfèkd dVky VhVeUV l d lkFk VLV VhVeUV l dh ryuk dju d fy, vkf'kd :i l lrfyr f}nyh; Cy,d ¼ihchchch½ fMtkbu d dN u, ljpuk fofèk çnku djrk gA

fofèk;k% leg foHkkT; l?k] vkj pØh; l?k tl l?k ;ktukvk ij vkèkkfjr vkf"kd :i l lrfyr vi.kZ Cy,d ¼ihchvkbch½ vfHkdYiuk dk bu ljpuk fofèk dk fodfl r dju d fy, mi;kx fd;k tkrk gA

ifj.kke% d"ky ihchchch vfHkdYiukvk dh ,d lph ekin.M eY;k v_1 ¼VLV VhVeUV l dh l[;k½ ≤10 v_2 ¼dVky VhVeUV l dh l[;k½ ¾2] r_1 ¼VLV VhVeUV l dh çfr—fr½ ≤10 vkj r_2 ¼dVky VhVeUV l dh çfr—fr½ ≤ 15 d fy, SAS dkM dk mi;kx djd x.kuk fd, x, fopj.k d lkFk "kkfey gA

"kCn dtk% leg foHkkT; vkf"kd :i l lrfyr f}i{kh; vfHkdYiuk] vkf"kd :i l lrfyr vi.kZ Cy,d vfHkdYiuk] vkf"kd :i l lrfyr f}nyh; Cy,d vfHkdYiuk] SAS l,¶Vo;j] vèk&fu;fer leg foHkkT; vfHkdYiukA

Incomplete Block Designs for Comparing Test Treatments with Multiple Controls

Vinayaka[1,2], B.N. Mandal[3], Rajender Parsad[4], L.N. Vinaykumar[1*], P. Murali[2], Amaresh[2], Shweta Kumari[2] **10.18805/BKAP723**

ABSTRACT

Background: This article provides some new construction methods of partially balanced bipartite block (PBBB) designs for comparing test treatments with more than one control.

Methods: Partially balanced incomplete block (PBIB) designs based on some association schemes such as group divisible association, and cyclic association are used for developing these methods of construction.

Result: A catalogue of efficient PBBB designs is included for parameter values v_1 (number of test treatments) ≤10, v_2 (number of control treatments) ≤10 and r_2 (replications of test treatments) and (replications of control treatments) ≤15 along with computed variances using SAS code.

Keywords: Group divisible partially balanced bipartite design, Partially balanced bipartite block design, Partially balanced incomplete block design, SAS software, Semi-regular group divisible design.

ifjp;

df'k] vk|kfxd vkj tfod ç;kxk tl ç;kxk d db {k=k e] ç;kxdrk vDlj ,d lkFk db VLV VhVeUV l dh ryuk ekud ;k ekStnk mipkj d lkFk djuk pkgrk g ftl fu;=.k mipkj dgr gA ,dy dVky fLFkfr d fy, cgr lkj lkfgR; miyCèk g] mnkgj.k d fy,] gnk;r ,o lg;kxh ¼1988½(etenkj ¼1996½(vkj xlrk o çlkn ¼2001½A bl fLFkfr d fy,] vfHkdYiukvk dk ,d egRoi.k ox ftl cpgkQj o rEgku ¼1981½ }kjk fn;k x;k g mlls lrfyr mipkj vi.kZ Cy,d ¼chVhvkbch½ vfHkdYiuk dgk tkrk gA bl dkn bu vfHkdYiukvk dk vè;;u etenkj o ukVt ¼1983½(ukVt o rEgku ¼1983½(fgnk;r o etenkj ¼1984½(

[1]The Graduate School, ICAR-Indian Agricultural Research Institute, Pusa, New Delhi- 110 012, India.
[2]ICAR-Sugarcane Breeding Institute, Coimbatore-641 007, Tamil Nadu, India.
[3]ICAR-Indian Agricultural Research Institute, Hazaribagh-825 405, Jharkhand, India.
[4]ICAR-Indian Agricultural Statistics Research Institute, Pusa, New Delhi-110 012, India.

Corresponding Author: L.N. Vinaykumar, The Graduate School, ICAR-Indian Agricultural Research Institute, Pusa, New Delhi-110 012, India. Email: vinaymandya123@gmail.com

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LVQdu ¼1987] 1988½(px ,o lg;kxh ¼1988½(çlkn ,o lg;kxh ¼1995½(nkl ,o lg;kxh ¼2005½(eMy ,o lg;kxh ¼2017½(vkj eMy ,o lg;kxh ¼2020½ }kjk fd;k x;k FkkA chVhvkbch vfHkdYiuk ge"kk lHkh ekin.M l;ktuk d fy, miyCèk ugh gk ldr g ;k ;gk rd fd vxj ;g miyCèk g] rk bl cMh l[;k e çfr—fr dh vko";drk gk ldrh gA bl leL;k dk njfdukj dju d fy,] tØ,Dl ¼1986½ u leg foHkkT; VhVeUVl ¼thMhVhMh½ vfHkdYiuk fn, gA bu vfHkdYiukvk ij vkx tØ,Dl ¼1987] 1988½(tØ,Dl o etenkj ¼1989½(vkj LVQdu ¼1991½ u vè;;u fd;k gA

gkykfd] ,d çk;kfxd fLFkfr ekt n g] ftle ç;kxdrk ,d l vfèkd fu;=.k d lkFk db VLV VhVeUVl dh ryuk dju e #fp j[krk gA bl fLFkfr d fy,] "k#vkr e] d,LVu ¼1962½ u viu lkf[;dh; fo"y"k.k d lkFk nk vyx&vyx l[;k e çfr—fr d lkFk lrfyr Cy,d vfHkdYiuk r;kj fd, gA ckn e] dkx;kek o flUgk ¼1988½ u vkipkfjd :i l ,d l vfèkd dVky VhVeUVl d lkFk db VLV VhVeUVl dh ryuk dju d fy, lrfyr f}nyh; Cy,d ¼chchihch½ vfHkdYiuk uked vfHkdYiukvk dk ,d eY;oku ox i"k fd;k vkj bu vfHkdYiukvk dh rkfydk, Hkh nhA bu vfHkdYiukvk dk ckn e flUgk vkj dkx;kek ¼1990½(tXxh ,o lg;kxh ¼1996½(vkj eMy ,o lg;kxh ¼2018½ d }kjk vè;;u fd;k x;kA chchihch vfHkdYiuk lHkh ekinMk d l;ktu d fy, ge'kk çkI; ugh gk ldrk g ;k ;fn ;g çkI; g rk Hkh cMh l[;k e çfr—fr dh vko";drk gk ldrh gA ,lh fLFkfr;k d fy,] jko ¼1966½ u "k: e fof"k'V çfr—fr vkj mud lkf[;dh; fo"y"k.k d lkFk vkf"kd :i l lrfyr Cy,d vfHkdYiuk, fodflr fd, gA ckn e] vfHkdYiukvk dk lcl mi;ä ox] ;kuh leg foHkkT; vkf"kd :i l lrfyr f}nyh; ¼thMh ihchch½ vfHkdYiuk igyh ckj dkx;kek o flUgk ¼1991½ }kjk çLrkfor fd;k x;kA dN vU; ,lkfl,"ku ;ktukvk ¼;kuh] pØh;] yfVu ox vkj f=dk.kh; ,lkfl,"ku ;ktukvk½ ij vkèkkfjr bu vfHkdYiukvk dk Hkh vè;;u fd;k x;k g ¼fouk;dk ,o lg;kxh] 2023½A bu vfHkdYiukvk d Qk;nk ij dkx;kek o flUgk ¼1988½ }kjk ppk dh xb gA dkx;kek o flUgk ¼1991½ d vuqlkj] ; vfHkdYiuk QDVkfj;y ç;kxk d lpkyu d fy, Hkh mi;kxh g(mnkgj.k d fy,] ijh o dkx;kek ¼1985½(vkj ijh ,o lg;kxh ¼1986½ n[kA bl vfHkdYiuk dk vxyh dMh e ifjHkkf"kr fd;k x;k gA

ifjHkk"kk

,d vi.k Cy,d vfHkdYiuk ftle ν_1 VhVeUVl dk ,d lV r_1 ckj vkj nljk ν_2 VhVeUVl dk lV r_2 ¼$r_1 \neq r_2$½ ckj ?kfVr gkrk g] k vkdkj d b [k.Mk e 0;ofLFkr gk] rk ml thMh ihchch vfHkdYiuk dgk tkrk g ;fn¼ ¼i½ jth lV d VhVeUVl dk p_j legk e foHkkftr fd;k tk ldrk g ftle çR;d leg dk vkdkj q_j (j=1,2) g] vkj ,d gh leg e fdlh Hkh nk VhVeUVl dk igyk lg;kxh dgk tkrk g] vU; Fkk nlj lg;kxh(¼ii½ jth lV e dkbZ Hkh nk VhVeUVl tk ith lg;kxh g vkj $\lambda_{j(i)}$ [k.Mk e ,d lkFk gkr g] tgk ij i¾1]2(g ¼iii½ fofHkUu lVk l dkbZ Hkh nk VhVeUVl λ_{12}¾λ_{21} ½>0½ [k.Mk e ,d lkFk gkr gA

çrhdk tl ν_1, ν_2, b, r_1, r_2, k, $\lambda_{j(i)}$ ¼j¾1]2(i¾1]2]...]t½] dk vfHkdYiuk d ekin.M dgk tkrk gA bl vfHkdYiuk dk vke rkj ij vkf"kd :i l lrfyr f}nyh; Cy,d ¼ihchchch½ vfHkdYiuk d :i e tkuk tkrk gA ;fn ihchchch vfHkdYiuk] leg foHkkT; ¼thMh½ l?k ;ktuk ij vkèkkfjr g] rk bl thMh ihchch vfHkdYiuk dgk tkrk gA bld vykok] ; vfHkdYiuk VhVeUVl d nk legk d Hkhrj vkf"kd lryu cukr g ¼çR;d leg e VhVeUVl dh fujrj çfr—fr gkrh g½ vkj lkFk gh legk d chp lryu lqpuk Hkh gkrh gA è;ku n fd λ_{12}= 0 gku ij] vfHkdYiuk e vlc)rk gk tkrk g blfy, ge λ_{12}>0 çfrcèk nr gA bld vykok] tc $\lambda_{2(1)}$ ¾$\lambda_{2(2)}$] bl $\lambda_{2(\cdot)}$ }kjk n"kk;k tkrk gA ihchchch vfHkdYiuk d fy, fuEufyf[kr ekin.M d lcèk ykx gkr g%

$$\text{(a). } \nu_1 r_1 + \nu_2 r_2 = bk$$

$$\text{(b). } r_1\,(k\text{-}1) = n_{11}\lambda_{1(1)} + n_{12}\lambda_{1(2)} + \nu_2\lambda_{12}$$

$$\text{(c). } r_2\,(k\text{-}1) = n_{21}\lambda_{2(1)} + n_{22}\lambda_{2(2)} + \nu_2\lambda_{12}$$

;fn $\lambda_{2(1)}$ ¾ $\lambda_{2(2)}$ rc ¼c½ dk r_2 ¼k&1½¾¼ν_2&1½ $\lambda_{2(\cdot)}$$ $\nu_1\lambda_{12}$ e ldfpr fd;k tkrk gA ;gk n_{ji}] jth ¼j¾1] 2½ lV l fn, x, VhVeUVl d ith ¼i ¾ 1] 2½ lg;kfx;k dh l[;k gA

y[k dk fuEukuqlkj 0;ofLFkr fd;k x;k gA èkkjk 2 lokÙke ekunM vkj ihchchch vfHkdYiukvk dh fopj.k lajpuk d ckj e tkudkjh çdV djrh gA bu vfHkdYiukvk dh db ubZ fuek.k fofèk;k fp=.k d lkFk èkkjk 3 e nh xb gA ifjf"k'V B e x.kuk fd, x, fHkUurkvk ¼tkSAS l,¶Vo;j dk mi;kx djd fuèkkfjr x; g½ d lkFk vfHkdYiuk dh ,d lph nk≶kxh oxk ekin.M d lkFk tk $\nu_1 \le 10$, $\nu_2 = 2$, $r_1 \le 10$, $r_2 \le 15$ g] mUg Hkh fn;k x;k gA ;g y[k èkkjk 4 e ,d Bkl fu'd'k d lkFk lekIr gkrk gA

lkexh vkj fofèk;k

ç;kxdrk dh #fp çR;d dVky VhVeUVl d lkFk çR;d VLV VhVeUVl d chp ;Fkklaho mPp ifj'k)rk d lkFk çkFkfed fojkèkkHkklk dk vueku yxku e gA VLV cuke dVky ryuk dju d fy, [k.M vfHkdYiuk d b'Vre igyvk ij lkfgR; e miyCèk ifj.kkek dk mi;kx djd vfHkdYiuk dh n{krk çkIr dh tk ldrh gA VLV cuke dVky ryuk dju d fy, d"ky ¼b'Vre½ vfHkdYiuk dh [kkt d fy, lcl mi;ä ekunM A&b'Vrerk gA A&b'Vre vfHkdYiuk] vfHkdYiukvk d çfrLièkÊ ox D e çR;d dVky VhVeUV l d lkFk VLV VhVeUV l d çkFkfed fojkèkkHkkl k d vkl r fHkUurk dk de djrk gA ,dy fu;=.k fLFkfr;k d fy,] vfHkdYiuk dk çfrLièkÊ ox D_1 ¼v,b,k½ g] tk dh duDVM Cy,d vfHkdYiukvk dk ox g ftle v VLV VhVeUV l vkj dVky VhVeUV l çR;d k vkdkj d b [k.M e 0;ofLFkr fd, tkr gA ,d l vfèkd fu;=.kk d fy,] vfHkdYiuk dk çfrLièkÊ ox $D_2\,(v_1,v_2,b,k)$ g] tk dh duDVM Cy,d vfHkdYiuk dk ox g ftle v_1 VLV VhVeUV l vkj v_2 dVky VhVeUV l çR;d k vkdkj d b [k.M e 0;ofLFkr gkr gA oreku tkp e] ,d l vfèkd dVky VhVeUV l d fy,] mu vfHkdYiukvk ij tkj fn;k tk,xk tk VLV cuke dVky ryuk d fy, vkf"kd :i l lrfyr gA vxyh dMh e] çR;d dVky VhVeUV l d lkFk çR;d VLV VhVeUV l dh ryuk dju d fy, ihchchch vfHkdYiuk d fy, b'Vrerk ekunM ifjHkkf"kr fd, x, gA

çkIr ihchchch vfHkdYiukvk dh n{krk dk vè;;u dju d fy,] ge ekur g fd $D(v_1,v_2,b,k)$ lHkh tM g, [k.M vfHkdYiukvk dk ox g ftle v_1 VLV VhVeUV l vkj v_2 dVky VhVeUV l çR;d k vkdkj d b [k.M e 0;ofLFkr fd, tkr gA geu fouk;dk ,o lg;kxh ¼2023½(}kjk ppk dh xb ihchchch vfHkdYiukvk dh A&b'Vrerk LFkkfir dju d fy, i;kIr fLFkfr dk mi;kx fd;k gA i;kIr fLFkfr lHkh VLV VhVeUV l cuke dVky VhVeUV l fojkèkkHkkl k d fopj.k&lgçlj.k efVDl d fu"kku dk fupyh lhek nrh gA ,d vfHkdYiuk tk fupyh lhek dk çkIr djrk g ml A&b'Vre dgk tkrk gA "kr ifj.kke 2-1 e nh xb gA

ifj.kke 2-1% ihchchch vfHkdYiuk v_1] v_2] b] k d leku ekuk d lkFk lHkh vfHkdYiukvk dh J.kh e A&b'Vre g ;fn lcl cM i.kkd QD"ku dk n"kkrk g %

tgk]

$$g(w,q) = \min\{g(x,z),\ (x,z) \in \Delta\} \qquad (2.1)$$

$\Delta=\{(x,z);\ x=0,1,..\text{int}\{k/v_2\}-1;\ z=0,1..,b,\ z>0$, d lkFk] tc $x=0\}$

$$g(x,z) = \frac{1}{A(x,z)} + \frac{a}{B(x,z)} + \frac{d}{C(x,z)},$$

$$a = v_2(v_1-1)^2,\ d = v_1(v_2-1),$$

$$A(x,z) = \{k(bx+z) - v_2(bx^2+2xz+z)\}/v_1 k,$$

$$B(x,z) = [bkv_1(k-1) - v_2(v_1(k-1)+k)(bx+z) + v_2^2(bx^2+2xz+z)]/v_1 k,$$

$C(x,z) = bx+z$. ;gk int[.] lcl cM i.kkd QD"ku dk n"kkrk gA

;gk ihchchch vfHkdYiukvk dh A-n{krk ¼E½ çkIr dju d fy, LVQdu ¼1988½ d –f"Vdk.k dk viuk;k gA A&n{krk ,d dkYifud A-b'Vre vfHkdYiuk d A&eY; dk vuikr g ftldk ekunM eku ¼2-1½ e fn;k x;k g] vfHkdYiukvk d fdlh fn, x, ox e dVky VhVeUV l d lkFk VLV VhVeUV dh ryuk dju d fy, U;ure g] vfHkdYiuk d A&eY; d fy, ftldk A&n{krk ,d gh J.kh d vfHkdYiukvk e çkIr dh tkuh gA ;gk] A& eY; vif{kr VhVeUV l fojkèkkHkkl k d fopj.k&lgçlj.k efVDl dk vuj[k gA bu vfHkdYiukvk dh A&n{krk, ifj.kke 2-1 dk mi;kx djd çkIr dh tkrh gA bld vykok] ;fn A&n{krk dk eku 1-000 g rk ,d vfHkdYiuk A&b'Vre g ¼fouk;dk ,o lg;kxh] 2024½A ekin.M $v_1 \le 10$, $v_2=2$, $r_1 \le 10$ vkj $r_2 \le 15$ d nk;js e dN A&b'Vre vfHkdYiuk ifjf"k'V e n[k x, gA lphc) dju d mí"; l] geu v_2 ¼dVky VhVeUV l dh l[;k½ ¾ 2 rd lhfer j[kk gA bld vykok] nk VhVeUV l] $V_{1(1)}\sigma^2$, $V_{1(2)}\sigma^2$, $V_{12}\sigma^2$ vkj $V_{2()}\sigma^2$ d chp çHkko e vuekfur vrj d fy, bu vfHkdYiukvk dh x.kuk dh xb çlj.k Hkh fn;k x;k gA ;gk] $V_{1(1)}\sigma^2$ igy lV l nk VLV VhVeUV l ¼igy lg;kfx;k½ d chp çHkko e vuekfur vrj d fy, fHkUurk dk n"kkrk g($V_{1(2)}\sigma^2$ igy lV l nk VLV VhVeUV l ¼nlj lg;kfx;k½ d chp çHkko e vuekfur vrj d fy, fHkUurk dk n"kkrk g($V_{2()}\sigma^2$ nlj lV l nk dVky VhVeUV l d fy, fHkUurk dk n"kkrk g(vkj $V_{12}(=V_{21})\sigma^2$ fofHkUu lVk l nk VhVeUV l ¼dVky d lkFk VLV½ d fy, fHkUurk dk n"kkrk gA ; ek=k, $V_{1(1)}$, $V_{1(2)}$, V_{12} vkj $V_{2()}$;g fuèkkfjr dju e Hkh lgk;d g fd vfHkdYiuk 0;kogkfjd g ;k ughA ç;kxdrk vkj vH;kl dju okyk lkf[;dhfon fn, x, v_1] v_2 vkj b d fy, eku $V_{1(1)}$, $V_{1(2)}$, V_{12} vkj $V_{2()}$ i<dj rkfydkvk l viu 0;kogkfjd mí"; d fy, mi;ä vfHkdYiuk dk p;u dj ldr gA ifjf"k'V e v{kj S, SR, R, LS, T, C DyVoFkÊ ¼1973½ l lkr vfHkdYiuk n"kkr gA

ifj.kke vkj ppk

ihchchch vfHkdYiukvk dh fuek.k fofèk;k

;g vuHkkx dN lkekU; ,lkfl,"ku ;kstukvk d vkèkkj ij ihchvkbch vfHkdYiuk dk mi;kx djd ihchchch vfHkdYiuk çkIr dju d fy, dN u, rjhdk dk [kykl k djrk gA ihchvkbch vfHkdYiuk d vfèkd fooj.k d fy,] ijlkn ,o lg;kxh ¼2007a, b½; oxÊ l ,o lg;kxh ¼2020½(vkj fouk;d o ilkn ¼2023½ dk i< ldr gA

lajpuk fofèk 1

eku yhft , fd fdlh ,lkfl,"ku Ldhe ij vèkkfjr ,d t&,lkfl,V Dykl ihchvkbch vfHkdYiuk g ftldk ekin.M v′, b′, r′, k′ = 3, λ'_i, i=1,2…t g vkj bld j[th] [k.M ¼x_{1j}, x_{2j}, x_{3j}½] j=1,2,...,b′ gA çR;d b′ [k.M e v_2 fu;=.k $0_1, 0_2, \ldots, 0_{v_2}$] tkM tlk dh uhp fn;k x;k gA vc lj.kh ¼x_{1j}, x_{2j}, x_{3j} $0_1, 0_2, \ldots, 2_{v_2}$) dk rhu [k.Mk e 0;ofLFkr dj¼ ($x_{1j}, x_{2j}, 0_1, 0_2, \ldots, 0_{v_2}$); ($x_{1j}, x_{3j}, 0_1, 0_2, \ldots, 0_{v_2}$) vkj ($x_{2j}, x_{3j}, 0_1, 0_2, \ldots, 0_{v_2}$)- ihchvkbch vfHkdYiuk d çR;d b′ [k.M d fy, bl çfØ;k dk nkgjku l v_1¾v'] v_2] b¾3b′] r_1¾2r'] r_2¾3b′] k¾2v_2] $\lambda_{1(i)}$ ¾λ'_i] λ_{12}¾2r'] $\lambda_{2()}$¾3b′ tl ekinMk d lkFk ,d ihchchch vfHkdYiuk çkIr gkrk gA

mnkgj.k 1% ,d vèk&fu;fer thMh vfHkdYiuk] ;kuh] DyVokFkÊ ¼1973½ SR18 ftldk ekinM v'= 6] b′¾ 4] r' ¾ 2] k′¾ 3] λ'_1¾ 0] λ'_2¾ 1] m¾ 3] n¾ 2 g bld [k.Mk dk uhp fn;k x;k g%

(1, 2, 3)
(1, 5, 6)
(2, 4, 6)
(3, 4, 5)

fofèk 1 dh çfØ;k d vulkj] çkIr ihchchch vfHkdYiuk d [k.M dh lajpuk bl çdkj g%

$(1, 2, 0_1, 0_2)$; $(2, 4, 0_1, 0_2)$
$(1, 3, 0_1, 0_2)$; $(2, 6, 0_1, 0_2)$
$(2, 3, 0_1, 0_2)$; $(4, 6, 0_1, 0_2)$
$(1, 5, 0_1, 0_2)$; $(3, 4, 0_1, 0_2)$
$(1, 6, 0_1, 0_2)$; $(3, 5, 0_1, 0_2)$
$(5, 6, 0_1, 0_2)$; $(4, 5, 0_1, 0_2)$

vfHkdYiuk d ekinM v_1¾6] v_2¾2] b¾12] r_1¾4] r_2¾12] k¾4] $\lambda_{1(1)}$¾0] $\lambda_{1(2)}$¾1] λ_{12}¾4] $\lambda_{2()}$¾12 gk tkr gA $v_1 \leq 10$] v_2¾2] $r_1 \leq 10$ vkj $r_2 \leq 15$ d fy, fofèk 1 }kjk r;kj fd, x, vfHkdYiuk ifjf"k'V A dh rkfydk 1 e lphc) gA

lajpuk fofèk 2

eku yhft , fd ,d t&,lkfl,V Dykl ihchvkbch vfHkdYiuk g ftldk ekin.M v'] b′] r′] k′¾3] λ'_i] i¾1]2]…]t g vkj bld j[th] [k.M ¼x_{1j}, x_{2j}] x_{3j}½] j¾1]2]…]b^ gA çR;d b′ [k.M e v_2¾2 $ 0_1] vkj 0_2 tkMA lkj.kh ¼x_{1j}, x_{2j}] x_{3j}] 0_1]0_2½ dk pkj [k.Mk e 0;ofLFkr dj% ¼x_{1j}]0_1]0_2½(¼x_{2j}]0_1]0_2½(¼x_{3j}]0_1]0_2½(vkj ¼x_{1j}]x_{2j}]x_{3j}½- ihchvkbch vfHkdYiuk d çR;d b^ [k.M d fy, bl çfØ;k dk nkgjku l v_1¾v'] v_2¾2] b¾4b′] r_1¾2r′] r_2¾3b′] k¾3] $\lambda_{1(i)}$¾λ'_i] λ_{12}¾r'] $\lambda_{2()}$¾3b′ tl ekinMk d lkFk ,d ihchchch vfHkdYiuk dk irk pyrk gA

mnkgj.k 2% ,d vèk&fu;fer thMh vfHkdYiuk SR18 ¼DyVokFkÊ] 1973½ e ftldk ekinM v'¾6] b′¾4] r'¾2] k'¾3] λ'_1¾0] λ'_2¾1] m¾3 vkj n¾2 gA fofèk 2 dh çfØ;k dk ykx djd] bu [k.M d lkFk ,d ihchchch vfHkdYiuk çkIr fd;k tk ldrk g%

$(1, 2, 3)$; $(2, 4, 6)$; $(1, 4, 5)$; $(3, 5, 6)$
$(1, 0_1, 0_2)$; $(2, 0_1, 0_2)$; $(1, 0_1, 0_2)$; $(3, 0_1, 0_2)$
$(2, 0_1, 0_2)$; $(4, 0_1, 0_2)$; $(4, 0_1, 0_2)$; $(5, 0_1, 0_2)$
$(3, 0_1, 0_2)$; $(6, 0_1, 0_2)$; $(5, 0_1, 0_2)$; $(6, 0_1, 0_2)$

bl vfHkdYiuk d ekin.M g v_1¾6] v_2¾2] b¾16] r_1¾4] r_2¾12] k¾3] $\lambda_{1(1)}$¾0] $\lambda_{1(2)}$¾1] λ_{12}¾2] $\lambda_{2()}$¾12 gA fofèk 2 }kjk mRiUu vfHkdYiukvk dk ifjf"k'V A e vkj $v_1 \leq 10$] v_2¾2] $r_1 \leq 10$ o $r_2 \leq 15$ d fy, rkfydk A2 e lphc) fd;k x;k gA

lajpuk fofèk 3

eku yhft , fd ,d t-,lkfl,V Dykl ihchvkbch vfHkdYiuk g ftldk ekin.M v′, b′, r′, k′ = 2, λ'_i, i=1,2…t g vkj bld j[th] [k.M ¼x_{1j}, x_{2j}½] j¾1]2]…]b′ gA çR;d b′ [k.M e v_2 dVky VhVeUV l 0_1]0_2]…]0_{v_2} tkMA lkj.kh ¼x_{1j}, x_{2j}]0_1]0_2]…]0_{v_2}½ dk ¼2$v_2$$1½ [k.Mk e 0;ofLFkr dj tl fd% ¼x_{1j}]0_1½] ¼x_{1j}]0_2½]…] ¼x_{1j}]0_{v_2}½(¼x_{2j}]0_1½] ¼x_{2j}]0_2½] …] ¼x_{2j}]0_{v_2}½ vkj ¼x_{1j}]x_{2j}½- ihchvkbch vfHkdYiuk d lHkh b′ [k.M d fy, bl çfØ;k dk nkgjku l v_1¾v′] v_2] b¾¼2$v_2$$1½b′] r_1¾¼$v_2$$1½r'] r_2 ¾ 2b′] k¾2] $\lambda_{1(i)}$¾λ'_i] λ_{12}¾r'] $\lambda_{2()}$¾0 tl ekinMk d lkFk ,d ihchchch vfHkdYiuk dk irk pyrk gA

mnkgj.k 3% eku yhft; ,d ihchvkbch vfHkdYiuk g tk vèk&fu;fer thMh vfHkdYiuk g ftldk ekinM v_1¾4] b'¾4] r'¾2] k'¾2] λ'_1¾0] λ'_2¾1] m¾2 vkj n¾2 [DyVokFkÊ ¼1973½ e SR1] gA bldk [k.M vfHkU;kl vkx fn;k x;k g%

rkfydk A1: fofèk 1 dk mi;kx djd $v_2 \leq 10$, v_2¾2, $r_1 \leq 10$, $r_2 \leq 15$ d fy, ihchchch vfHkdYiukA

S.N.	v_1	v_2	b	r_1	r_2	k	$\lambda_{1(1)}$	$\lambda_{1(2)}$	λ_{12}	$\lambda_{2()}$	$V_{1(1)}$	$V_{1(2)}$	V_{12}	$V_{2()}$	E	Source
1	5	2	15	6	15	4	2	1	6	15	0.40	0.42	0.27	0.13	1.00	C12
2	6	2	12	4	12	4	0	1	4	12	0.67	0.62	0.39	0.17	1.00	SR18

rkfydk A2% fofèk 2 dk mi;kx djd $v_1 \leq 10$] v_2¾2] $r_1 \leq 10$] $r_2 \leq 15$ d fy, ihchchch vfHkdYiukA

S.N.	v_1	v_2	b	r_1	r_2	k	$\lambda_{1(1)}$	$\lambda_{1(2)}$	λ_{12}	$\lambda_{2()}$	$V_{1(1)}$	$V_{1(2)}$	V_{12}	$V_{2()}$	E	Source
1	5	2	20	6	15	3	2	1	3	15	0.43	0.46	0.31	0.13	0.94	C12
2	6	2	16	4	12	3	0	1	2	12	0.75	0.68	0.45	0.17	0.96	SR18

rkfydk 3% fofèk 3 dk mi;kx djd $v_1 \leq 10$] v_2¾2] $r_1 \leq 10$] $r_2 \leq 15$ d fy, ihchchch vfHkdYiuk,A

S.N.	v_1	v_2	b	r_1	r_2	k	$\lambda_{1(1)}$	$\lambda_{1(2)}$	λ_{12}	$\lambda_{2()}$	$V_{1(1)}$	$V_{1(2)}$	V_{12}	$V_{2()}$	E	Source
1	4	2	20	6	8	2	0	1	2	0	0.67	0.58	0.48	0.50	0.83	SR1
2	5	2	25	6	10	2	1	0	2	0	0.59	0.68	0.45	0.40	0.85	C1

ifjf"k'V B: ihchchch vfHkdYiuk d dI;fVx çlj.k d fy, SAS dkMA

```
proc iml;
PBBBD={
1     2     5     6     ,
3     4     5     6     ,
.
.
.
2     3     5     6
}; /*Arrangement of treatments in m × n array*/
mu=j(nrow(PBBBD)*ncol(PBBBD),1,1);/*mean (mu) vector*/
/*print mu;*/
print PBBBD;
trt=j(nrow(PBBBD)*ncol(PBBBD),max(PBBBD),0);/*Design matrix-observations versus
treatments*/
k=1;
do i=1 to nrow(PBBBD);
do j=1 to ncol(PBBBD);
if PBBBD[i,j]>0
then trt[k, PBBBD[i,j]]=1;
k=k+1;
end;
end;
/*print trt;*/
r=j(nrow(PBBBD)*ncol(PBBBD),nrow(PBBBD),0);/*Design matrix-observations versus rows*/
k=1;
do i=1 to nrow(PBBBD);
do j=1 to ncol(PBBBD);
if PBBBD[i,j]>0
then r[k,i]=1;
k=k+1;
end;
end;
/*print r;*/
c=j(nrow(PBBBD)*ncol(PBBBD),ncol(PBBBD),0);/*Design matrix-observations versus
columns*/
k=1;
do i=1 to nrow(PBBBD);
do j=1 to ncol(PBBBD);
if PBBBD[i,j]>0
then c[k,j]=1;
k=k+1;
end;
end;
/*print c;*/
x=mu||trt||r;/*Combined Design matrix*/
x1=trt;
```

(1, 2)
(3, 4)
(1, 4)
(2, 3)

vc fofèk 3 dh çfØ;k dk ikyu djr g,] v_2¾2 d fy,] ge [k.M d lkFk thMh lcfèkr ;ktuk ij vkèkkfjr ,d ihchchch vfHkdYiuk feyrk g¼

(1, 0_1); (3, 0_1); (1, 0_1); (2, 0_1)
(1, 0_2); (3, 0_2); (1, 0_2); (2, 0_2)
(2, 0_1); (4, 0_1); (4, 0_1); (3, 0_1)
(2, 0_2); (4, 0_2); (4, 0_2); (3, 0_2)
(1, 2); (3, 4); (1, 4); (2, 3)

mijkä vfHkdYiuk d ekinMg v_1¾4] v_2¾2] b¾20] r_1¾6] r_2¾8] k¾2] $\lambda_{1¼1½}$¾0] $\lambda_{1¼2½}$¾1] λ_{12}¾2] $\lambda_{2¼ ½}$¾0- fofèk 3 }kjk mRiUu vfHkdYiuk dk ifjf"k'V A dh rkfydk A3 e $v_1 \leq 10$] v_2¾2] $r_1 \leq 10$ vkj $r_2 \leq 15$ d fy, lphc) fd;k x;k gA

;gk] fofèk 1] 1-00 d cjkcj n{krk d lkFk A-b'Vre vfHkdYiuk çnku djrh g ¼rkfydk A1½A bld vykok] fofèk;k 2 vkj 3 u 0-83 l 0-96 rd dh n{krk d lkFk pkj vfrfjä vfHkdYiuk Hkh r;kj fd,A blfy,] gk#u ,o lg;kxh ¼2016½ }kjk çtuu ijh{k.kk d fy, vfHkdYiuk] djekdj ,o lg;kxh ¼2022½ }kjk vkf"kd :i l lrfyr 3&vfHkdYiuk] vkj fou;dekj ,o lg;kxh ¼2023½(}kjk vkf"kd :i l nkgjk, x, ihchvkbch vfHkdYiuk d LFkku ij ikèkk vkj tkuojk d ç;kxk dh okLrfod thou dh çk;kfxd fLFkfr;k e bu vfHkdYiukvk dk mi;kx fd;k tk ldrk g(tc dVky d lkFk VLV dh ryuk vR;r egRoi.k gA

fu'd'k

;g vè;;u ihchvkbch fMtkbuk dk mi;kx djd NkV [k.M vkdkj ¼;kuh] $2 \leq k \leq 2v_2$½ d lkFk ihchchch vfHkdYiuk çkIr dju d u, –f'Vdk.k çLrr djrk gA bld vfrfjä] x.kuk dh xb n{krkvk d lkFk bu vfHkdYiukvk dh ,d rkfydk Hkh çnku dh xb gA SAS l,¶Vo;j] "kkèkdrkvk dh fof"k'V vko";drkvk dk ijk dju d fy, bu vfHkdYiukvk dh fHkUurk ljpuk dh x.kuk dh lfoèkk çnku djrk gA ç;kxdrk vkj vH;kl dju oky lkf[;dhfon çnku fd, x, ekinMk d fy, fHkUurk eY;k vkj n{krkvk dh x.kuk djd rkfydkvk l viuh 0;kogkfjd vko";drkvk d fy, mi;ä vfHkdYiuk dk p;u dj ldr gA ;g "kkèk çk;kfxd ifj–";k d fy, egRoi.k g ftle VLV VhVeUV l vkj dVky VhVeUV l d chp ryuk dh tkrh g] tk –f'k] ikfjfLFkfrdh] m|kx] i"kikyu vkj fpfdRlk tl fofHkUu {k=k e d"ky lekèkku i"k djrk gA ; vfHkdYiuk cuku e ljy] ;Fkkfpr vkj mPp n{krk çnf"kr dju oky g] tk ç;kxdrkvk dk 0;kogkfjd vuç;kxk d fy, bUg viuku d fy, çkRlkfgr djrk gA

lUnHk

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