



feJ.k ç; kxkadsfy, I pñvM vfHkdYi ukvædk fuekZk

jkgg cutñ I hek tXxñ, Ynksoxñ¹ viZk Hæed] vfufHrk nÜk] fl uh oxñ

Hkk—vuq; &Hkkjrh; —f'k I kf[; dh vuq U/kku I æFkk] ykbcjh, oð; j i w k&110 012] ubZ fnYyh] Hkkj rA

i klr%Qjojh 2021

Lohdr% tu 2021

I kjæk

feJ.k ç; kx okLrfod thou dsç; kxkaeacgr vke gA feJ.k ç; kxkadsfy, vfHkdYi uk fodfl r djrs oDr feJ.k ?kVdka ds vuq kr dk p; u bl rjg I sf d; k tkrk gSfd xf.krh; e, My dks i; klr : i I sf QV fd; k tk I ds, oa eki nMka dk vuq kr yxk; k tk I dA —f'k ç; kxkae feJ.k ?kVd ykxwbui v/ ds dbZ I kr gks I drsg vFkok bui v/ dks Ql y ds vyx vyx of) pj.kkaeabl dnj foHkkftr fd; k tk I drk gSfd Ql y ij ykxwbui v/ dh dgy ek=k fLFkj gA feJ.k ç; kxkadsfy, dqky vfHkdYi uk; ami; kxh gksrsga tc çfrfØ; k feJ.k eaekst m vo; okads I ki şk vuq kr ij fuHk] gkrh gA I kfgR; eadbl, Yxksjne, oañij fLVDI mi yC/k gA yfdu feJ.k ç; kxkadsfy, , Yxksjne ds mi; kx eadkQh I hfer dke mi yC/k gA , Yxksjnfed —f"Vdksk dk mi; kx djds vyx & vyx vuq kr dsfy, , d fof'k"V e, My dsfy, Nk/h jUl ea feJ.k ç; kxkadsfy, vfHkdYi uk fodfl r djus dh vko'; drk gA bl v/; ; u ea geus feJ.k ç; kxkaeal pñvM vfHkdYi uk ds fuekZk dsfy, , Yxksjne fodfl r fd; k gA , Yxksjnfed —f"Vdksk i kja fjd —f"Vdksk dh rgyuk eadbl fca n/ka ts sf d fQV fd, tkusokyse, My(ju vkfn ds I nHkZ ea vfHkdYi uk fuekZk ea vf/kd cgrj , oal æe I kfc r gkrk gA ; svfHkdi uk; sokLrfod thou ç; kxkaeacgr vPNh rjg I svuqhy gA feJ.k ç; kxkadsfy, vfHkdYi ukvæds fuekZk ea, Yxksjne dk mi; kx u dgy dEI; Wskuy ykxr dks de djrk gS çfyd , d fujrj vfHkdYi uk fXM ea vfHkdYi uk dh vf/kd dqky [kkst Hkx djrk gA

'Kn dñH, Yxksjne —f"Vdksk] feJ.k vfHkdYi uk] feJ.k ç; kx] I pñvM vfHkdYi ukA

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Construction of Saturated Designs for Mixture Experiments

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ABSTRACT

Mixture experiments are very common in real life experiments. Designing a mixture experiment involves selection of the proportion of the mixture components in a fashion such that a mathematical model can be fitted adequately and the parameters could be estimated. In agricultural experiments, the mixture components may be several sources of the input applied or input may be applied at different crop growth stages in splits such that total quantity applied to the crop is constant. Efficient designs for mixture experiments are useful when the response is assumed to depend on the relative proportions of the ingredients present in the mixture. A number of algorithms and heuristics are available in

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¹Hkk—vuq; & dñæh; I æph ekfRL; dh vuq U/kku I æFkk] dñph&682 018] dñ y] Hkkj rA

literature; however, a limited work has been done in the use of algorithms for mixture experiments. There is a need to develop designs for mixture experiments in smaller number of runs for a specific model for varying proportions using algorithmic approach. In this study we have developed algorithms to construct saturated designs for mixture experiments. The algorithm provides a greater flexibility in design construction in comparison to the traditional approach in terms of models to be fitted; number of runs to be required etc. These designs are very well suited in real life experiments. The use of algorithms in construction of designs for mixture experiments not only reduces the computational cost but also results in a more efficient search of the design in a continuous design space.

Key words: Algorithmic approach, Mixture designs, Mixture experiment, Saturated designs.

çLrkuk

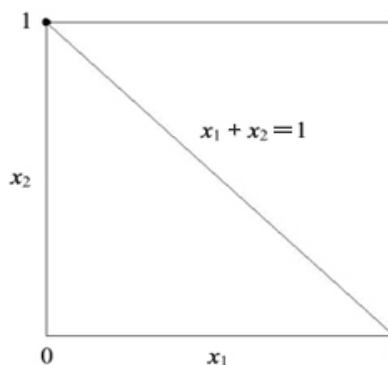
feJ.k ç; kx vke rkš ij vksj kfxd mRikn l=hdj.k ea ik, tkrš gš mnkgj.kkFk% [kk| çl djd.k] jkl k; fud l=hdj.k] ol= m| kx| vksk/kh; l=hdj.k bR; kfn] yfdu —f'k vuq dku ds dbZ {k=kaeafeJ.k ç; kxka dk bLreky l hfer gš vksj kfxd vuq dku ds l eku] —f'k vuq U/kku eaHkh feJ.k ç; kx dbZ {k=kaeabLreky djuseami; eš gš mnkgj.kkFk% mojdka; k dhVuk'kdka ds foHkuu feJ.kka dk , d Ql y ds iškokj ea çHko —f'k vuq U/kku ea dbZ fLFkr; k; ekStm gksl drh gStgka, d l exzfeJ.k çfrfØ; k i kjä fj d 0; fäxr çfrfØ; kvadh rgyuk eavf/kd mi; kxh gStš sf d] ekukdYpj cuke eVhdYpj [krh] cht feJ.k] enk feJ.k] [kk| feJ.k dk tkuoja ea çHko bR; kfn] bu l kjh i fj fLFkr; kaeafeJ.k fo'yšk.k rduhdka d 0; kogkfj d egRo gksl drk gš

, d feJ.k ç; kx eš Lora= dkjd , d feJ.k ds foHkuu ?kVdka ds vuq kr gkrš gš mnkgj.k ds fy,] ; fn vki LVuyd LVhy dh rU; rk rkdr dks vuq kr djuk pkgrš gš rksç; k t dsdkjd feJ.k /kkreaykšk] rkak] fudy vksj Økfe; e ds vuq kr gksl drs gš feJ.k ç; kxka eš çfrfØ; k fl QzfeJ.k eavo; oka; k ?kVdka d ki šk vuq kr ij fuHkj djrh gš vksj feJ.k dh ek=k ij ughš feJ.k dh ek=k dksç; kx ea, d vfrfjä dkjd ds: i eaHkh v/; ; u fd; k tk l drk gš ; g rF; fd foHkuu dkjdka ds vuq kr ea 100% ; kx gksuk pkfg,] vřHkdYi uk dsl kFk&l kFk feJ.k ç; kxka ds fo'yšk.k dks tfVy cukrk gš

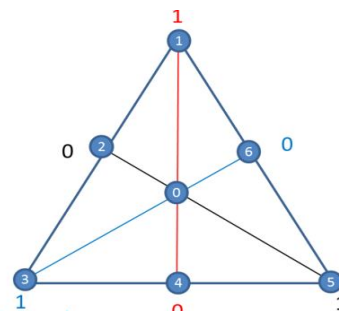
feJ.k ç; kxka dh fo'škkrk bl rF; l s gkrh gš fd çfrfØ; k y çR; d ?kVd Xi dk l ki šk vuq kr ij fuHkj djrh gš eku yHft, fd feJ.k ea ?kVd gš çR; d ?kVd 'kd; l s, d dschp eku ys l drk gš

$$\mathbf{x} = (x_1, x_2, \dots, x_q): x_i \geq 0, i = 1(1)q; \sum_{i=1}^q x_i = 1$$

; s ck/kk, a, d $\frac{1}{q}-1\frac{1}{2}$ vk; keh fl Ei ydI $\frac{1}{6q}-1\frac{1}{2}$ dks i fjHkk'kr djrh gš mnkgj.k ds fy,] ; fn feJ.k eanš ?kVd gš rks fl ElydI , d l h/kh jšk $\frac{1}{q}$ —fr $1\frac{1}{2}$ cu tkrh gš ; fn feJ.k earhu ?kVd gš rks fl ElydI , d l eckgq f=Hkq: $\frac{1}{q}$ —fr $2\frac{1}{2}$ cu tkrh gš



vk—fr 1% nks ?kVdh; feJ.k ç; kx $\frac{1}{q}x_1 + x_2 = 1\frac{1}{q}$



vk—fr 1% nks ?kVdh; feJ.k ç; kx $\frac{1}{q}x_1 + x_2 + x_3\frac{1}{q}$

I kfgR; eadbfJ.k vfHkdYiuk; aekStm gSftuesl cl s vke g%

- fl EiyDI ySVI vfHkdYiukA
- fl EiyDI l b/jkbM vfHkdYiukA
- , fDI ; y vfHkdYiukA
- , DI Vhe oVDI vfHkdYiukA

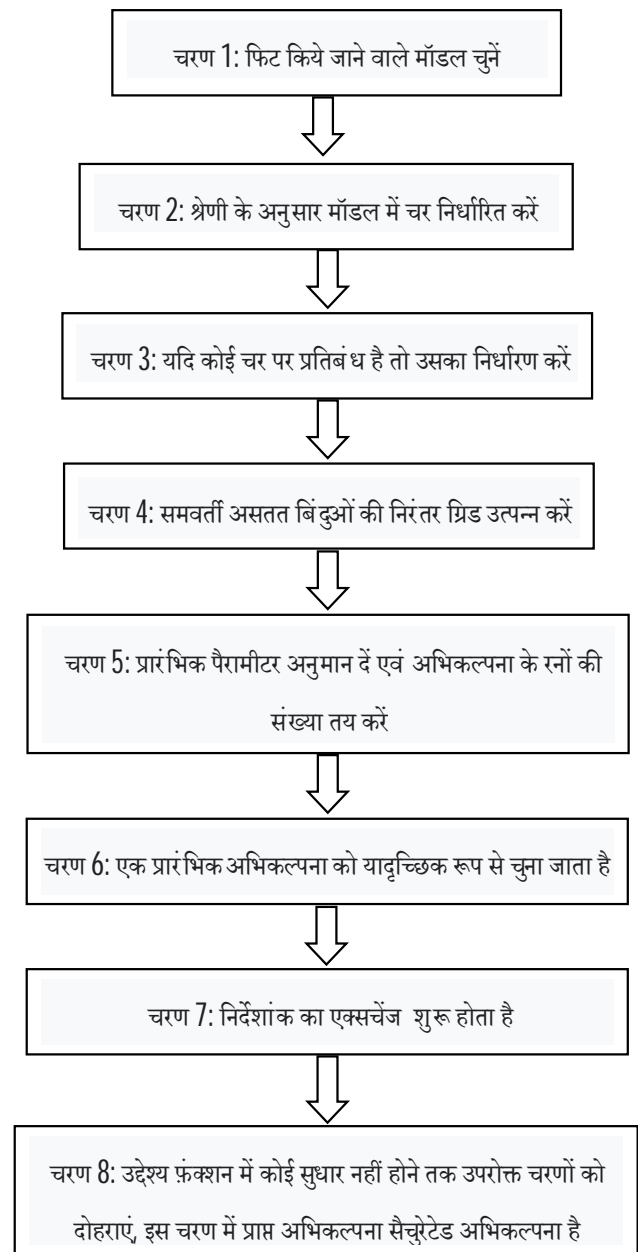
'kQs%1958½ usfl byDI ySVI vfHkdYiukvka dsl kFk mi ; kx fd, tkus okys dskudy i, yhvksu; El dks i sk fd; kA ; scgñ cfrck $\Sigma x_i = 1$ dk mi ; kx djds x_i ea l keW; cgn e, My dks l a kks/kr djdscklr fd, tkrsgA xj j[kh; e, My dk mi ; kx djus dk ykHk ; g gSfd] xj j[kh; ?kVuk dk cñ'kz djusokyh fLFkr; kadsekeys e] xj j[kh; e, My j[kd e, My dh ryuk eafj'rs dks cgrj < k l sl e > k, kA xj j[kh; e, My eafj[kd e, My dh ryuk eal eku feJ.k / cfØ; k dh l eL; k dk vuoku yxkus ds fy, de i jkehVj gkrs gA feJ.k ç; kxka ea vfHkdYiuk fueZk ea, Yxkfjfed -f'Vdsk fuEufyf[kr fcmvkaekStm rduhd l stknk dkjxj l kfor gksl drk gA

- vfHkdYiuk dsfy, jukadh l ; k pouseA
- feJ.k ç; kxkaeal kexh dsfy, vuqkr dk p; u djuseA
- fofHku e, Mykadh fQVx dsvuq kj vfHkdYiuk fueZk ea
- vfHkdYiuk ekumka dks i jk djus dsfy, vfHkdYiukvka ds fueZk ea

I kfgR; eaygysl sMveDI , YxkfjFke 'fe'ky 1974½ ds , y & , DI pat , YxkfjFke ¼ Vfdl u , oankus 1989½ XVERT , YxkfjFe ¼ h , oa ekjDokM 1974½ , oa XVERT1-0 , YxkfjFke Vuxe 1979½ 'kfey g] yfdu] , Yxkfjnfed -f'Vdsk dk mi ; kx djds vyx&vyx vuqkr dsfy, , oa , d fof'kV e, My ds vuq kj Nksh l ; k eafeJ.k ç; kxka dsfy, vfHkdYiuk fodfl r djus dh vko'; drk gA bl fLFkr dseisutj geusbl v/; ; u ea geus, d , YxkfjFe fodfl r djus dh dks'k'k dh gS ftl l sge de l sde ju eafeJ.k ç; kxka dsfy, Nksh vfHkdYiuk; afodfl r dj l dA

i) fr

, d l pjsVM vfHkdYiuk dk vFkzgsfd e, My eavuekfur i jkehVj dh l ; k dscjkj vfHkdYiuk dk ju gkrs gA fodfl r , YxkfjFe dk o.ku l jy fkyo pkVZeaddn bl çdkj gA



vxj ge nks feJ.k vo; oka , oa rhu eki nMka ds l kFk , d feJ.k ç; kxRed fLFkr eku y]ftl sfuEufyf[kr e, My jkj n'kz k tk l drk g%

$$\eta = \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2; 0 \leq (x_1, x_2) \leq 1; x_1 + x_2 = 1$$

bl ç; kxRed fLFkr eaAj mfyf[kr , YxkfjFe dstfj; sge fofHku buØheM l kbt dsfy, , d rhu ju okyh l pjsVM vfHkdYiuk fodfl r dj l drsgs tks dh bl çdkj g%

buohel/ I kb t	dMMV/ I v/ dk vdkj	çlr I pgiVM vfhkdYi uk	
0-01	101	x_1	x_2
		0-5	0-5
		0-0	1-0
		1-0	0-0
0-02	51	x_1	x_2
		0-5	0-5
		1-0	0-0
		0-0	1-0
0-03	34	x_1	x_2
		0-49	0-51
		1-0	0-0
		0-0	1-0
0-04	26	x_1	x_2
		0-52	0-48
		1-0	0-0
		0-0	1-0
0-05	21	x_1	x_2
		0-5	0-5
		1-0	0-0
		0-0	1-0
0-06	17	x_1	x_2
		0-5	0-5
		1-0	0-0
		0-0	1-0
0-10	11	x_1	x_2
		0-5	0-5
		1-0	0-0
		0-0	1-0

fu"d"l

bl v/; ; u ea, d l a kks/kr , YxkfjFe dksçLrkfor fd; k
x; k gStksdh feJ.k ç; kxkaej\$[kd , oax\$ j\$kh; e, Myka
dsfy, l pgiVM vfhkdYi ukvka dk fuelzk djus ea l {ke
g\$; s vfhkdYi uk; sokLrfod thou ç; kxka dsfy, cgr

vuply g\$ çLrkfor , YxkfjFe dk vuç; kx dEl; w\$skuy
yxkr dksde djusvk\$ fujrj fMtkbu LFkku ea [kxst ds
ç; kl eaenn djrk g\$

I aHk

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