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Characterization of Lactation Curve Patterns using Non-linear Models in Crossbred Dairy Cattle

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Abstract

Background: Lactation curve patterns are currently integrated in dairy cow's management software. Lactation curve modeling is useful for monitoring individual yields for diet planning, determining optimum strategies for insemination and genetic evaluation. It also helps for predicting expected missing values on field records and gives concise summary of biological efficiency and persistency of dairy cows.

Methods: The study was aimed to characterize the lactation curve pattern for crossbred dairy cattle using different non-linear models. During the period 1991 to 2018, daily milk yield (DMY) consisted of 281698 records of 750 crossbred dairy cows maintained at Livestock Farms. GADVASU, Ludhiana, were collected for the study. Different non-linear models *viz.* exponential decline function (EDF), parabolic exponential model (PEM), inverse polynomial model (IPM), gamma-type function (GTF), mixed log function (MLF) and Ali and Schaeffer model (ASF) were used for the analysis. The model(s) that best fit and describe the curve characteristics was selected on the basis of coefficient of determination (R^2), coefficient of variation (CV), Akaike information criterion (AIC) and mean square error (MSE).

Result: The study clearly revealed that the PRM gave highest fit to DMY data with R^2, MSE, AIC and CV values of 98.10%, 0.087, -743.31 and 2.37%, respectively. The IPM had also best fitted the observed DMY data with highest R^2 (98.05%), lower MSE (0.089), low AIC (-735.8972) and lower CV (2.40%) values. The fitting of observed DMY data with predicted DMY were also found to be higher in the MLF (R^2= 96.46%, MSE= 0.159, AIC= -558.16 and CV= 3.21%) and GTF (R^2= 95.85%, MSE= 0.190, AIC= -505.24 and CV= 3.50%), whilst the EDF and PEM Models depicted relatively low fit to the DMY data when compared with the other non-linear models. However, IPM and GTF models can be used for accurate prediction of daily milk yield in the crossbred cattle population because they were typical standard lactation curves.

Key words: Crossbred dairy cattle, Daily milk yield, Lactation curve, Non-linear models.

Introduction

The graphical representation of milk production over the course of lactation in dairy cow is lactation curve. Costs of milk production depend on lactation yield and the persistency of lactation, which is an expression of the ability of the cow to continue to produce milk at peak level throughout lactation (Suresh *et al*., 2014). High persistency is associated with a slow rate of decline in milk production, whereas low persistency is associated with a rapid rate of decline in milk yield. Scott *et al*. (1996) reported that lactation curve provides valuable information about the pattern of milk production during lactation which is determined by the biological efficiency of the cow. Shobhana et al, 2016 studid about efficiency of various matemetical modles to predict the persistency of lacataion.

Several non-linear models such as Exponential decline function. Parabolic exponential model, Inverse polynomial model, the Gamma-type function Mixed log function and Polynomial regression function have been proposed recently to describe the lactation curve and modified by various investigators (Guo and Swalve 1995; Gengler 1996, Shobhana *et al.,* 2016) to circumvent computational difficulties. Gamma function has been to fit daily and/or weekly milk yield data of different crossbred cattle (Madalena *et al*., 1979). Inverse polynomial function has well explained the variation in weekly milk yield data of different cattle breeds (Kolte *et al*., 1986). Mixed Log Function has been used extensively in crossbred and in exotic dairy cattle. The mixed log function resulted in high goodness of fit for weekly milk yield data of different breeds such as Holstein Friesian (Olori *et al*., 1999) and Simmental cows (Cilek and Keskin 2008).

Lactation curve functions are currently integrated in dairy farm management software. At cow level, lactation curve modeling is of help for monitoring individual yields for

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diet planning, early detection of diseases before the appearance of clinical signs and for selecting animals to be culled, determining optimum strategies for insemination and genetic evaluation (Nicolς *et al.*, 2016). It also helps for predicting expected missing values on field records and gives concise summary of biological efficiency and persistency of dairy cow. Therefore, the present study was aimed to model lactation curve patterns and characteristics for crossbred dairy cattle using different non-linear models.

MATERIALS AND METHODS

Location

This study was performed on crossbred dairy cattle maintained at Directorate Livestock Farm (DLF) of Guru Angad Dev Veterinary and Animal Sciences University (GADVASU), Ludhiana, Punjab, India. Ludhiana is located at 30.9°N 75.85°E. It has an average elevation of 244 meters (798 ft) meters above mean sea level. It features a humid sub-tropical climate under the Koppen climate classification, with three defined seasons; summer (March to June), monsoon (July to September) and winter (October to February). The average high and low temperatures of the area were 29.8 and 16.7, respectively whereas the average maximum and minimum relative humidity were 82% and 46%, respectively. The district received annual average precipitation of 890 mm (Prabhiyot *et al*., 2013).

Herd Management Practices

The animals were housed under loose housing system. The advanced pregnant animals were segregated to the calving pens at least eight weeks before the expected date of calving. The calves were separately maintained in individual calf pens having adequate protection from adverse weather conditions and after four month of age they were shifted into loose housing system. The animals had free access to roughage feed and water. Feeding of the animals depended on the age and physiological status. Concentrate ration (CP=18% and TDN 70%) containing cereals, cakes, brans, mineral mixture, salt and additives was supplied as supplements.

Data Sets and Sources

Daily milk yield (DMY) consisted of 281698 records of 750 crossbred cows which were daughters of 125 sires and 537 dams; collected during the twenty-eight years (1991-2018) were used for the study. The data were obtained from daily milk yield registered sheets, maintained at Livestock Farms of Guru Angad Dev Veterinary and Animal Science University. Prior to analyses, daily milk yield data from sick, sold, culled, dead animals and cows having less than 200 days lactation length were removed from the study. Also, daily milk yield data produced from days one to five immediately post calving was not considered.

Statistical Analysis

Daily milk yield data of all animals, collected during the period of twenty-eight years were pooled averaged over three hundred five days and thereby fitted to different models (Table 1) *viz*. Exponential Decline Function (EDF), Parabolic Exponential Model (PEM), Inverse Polynomial Model (IPM), Gamma-Type Function (GTF), and Mixed Log Function (MLF). Polynomial regression function (PRM) was also used as described by Ali & Schaeffer 1987. The NLIN procedure of the Statistical Analysis Software version 9.3 (SAS institute Inc. 2011) was used to estimate the parameters of the models. The model(s) that best fit and describe the curve characteristics was selected based on goodness of fit statistics, namely coefficient of determination (R^2), Coefficient of variation (CV), Akaike information criterion (AIC), and mean square error (MSE). Residuals obtained by these models were plotted graphically. For Gamma-type function, the days in milk at peak yield (DIMP) was defined as b/c and the peak yield was estimated as: $a \times (b/c)^b e^{-b}$ whereas the persistency (P) of lactation was evaluated using: $P = -(b + 1) \ln(c)$ (Tekerli *et al*. 2000).

RESULTS AND DISCUSSION

Estimation of Lactation Curve parameters

The average daily milk yield was estimated as 12.57±0.01 kg for 281698 daily milk yield records of 750 crossbred cows. The average lactation curve parameters of crossbred cattle were estimated under various non-linear models (Table 2). The parameter 'a' was found to be positive and varied between 0.285 kg for the Inverse Polynomial Model to 16.336 kg for the EDF. Followed to EDF, PEM estimated high and positive 'a' parameter whereas the 'a' parameter values for GTF, MLF, and PRM were found to be 10.180±0.279, 7.213±0.292 and 5.106±0.276, correspondingly. Also, the parameter 'b' showed variation across the different non-linear models. The estimates of the parameter 'b' ranged from -1.444±0.025 (MLF) to 0.141±0.008 (GTF). Positive parameter 'b' was found in IPM and GTF with corresponding values of 0.0484±0.0003 and 0.141±0.008 while negative parameter 'b' was observed in the PEM, MLF and PRM which

Table 1: Equations of non-linear models used to describe the lactation of crossbred dairy cattle.

Models	Equations	References
Exponential decline function	$Y_t = ae^{-ct}$	Brody *et al.* 1923
Parabolic exponential model	$Y_t = a \exp(bt - ct^2)$	Sikka 1950
Inverse polynomial model	$Y_t = t(a + bt + ct^2)^{-1}$	Nelder 1966
The gamma-type function	$Y_t = at^b \times e^{-ct}$	Wood 1967
Mixed log function	$Y_t = a + b \times \sqrt{t} + c \times \log(t)$	Guo and Swalve 1995

(Where, Y_t = Average daily milk yield in the t^{th} day of lactation; a = initial milk yield after calving, b = ascending slope parameter up to the peak yield, c = descending slope parameter, t= length of time since calving and e_t= residual error) had values as 0.0004±0.0002, -1.444±0.025 and -0.097±0.0023, respectively. Similarly, differences were seen among the various non liner models fitted to the 305 days milk yield for the 'c' parameter values. In models having three parameters, 'c' parameter ranged from 4.84×10^{-6} $\pm4.81\times10^{-7}$ (PEM) to 4.668±0.120 (MLF). In the EDF, IPM, GTF and PRM, the 'c' parameter was positive and their corresponding values were $0.0018\pm4.3\times10^{-5}$, $0.000194 \pm 1.82\times10^{-6}$, $0.0031\pm7.4\times10^{-5}$ and $0.00013\pm4.742\times10^{-6}$ respectively. Parameters 'd' and 'f' in PRM were 4.240±0.099 and -6.523±0.1516, respectively and, showed increasing slope of lactation curve. For Gamma type function, the daily milk yield raised from calving to peak production of 15.145 kg reached on day 45th and then decreased gradually to dryness. The modelling of lactation curve provides guidelines in formulating farm managerial practices in dairy cows. The average initial daily milk yields (kg) estimated under all nonlinear models except EDF were low as compared to previous findings reported by Rekik *et al*. (2006), and Chegini *et al*. (2015). However average initial daily milk yields of all nonlinear models excluding IPM were higher than that found by Subham *et al*. (2017) for crossbred cattle. The average 'a' value estimated using EDF was also higher than that of Cole and Null (2009) who reported for dairy cattle under Wood model ('a'=13.01 kg). Occurrence of variations in the parameter 'a' value may be due to differences in genetic groups or in herd management.

The average b and c values in the IPM and GTF were in the scale of the previous studies as reported by Gradiz *et al*. (2009) and Chegini *et al*. (2015). The b parameter values in all nonlinear models were also lower than previous studies reported by Suham *et al*. (2017), Yogesh *et al*. (2017) for crossbred cattle. The c parameter values estimated under MLF was higher than reported by Suham *et al*. (2017) and Yogesh *et al*. (2017) for crossbred cattle. The production at the peak estimated under GTF was low compared to the study by Chegini *et al*., (2015) and Khalifa *et al*. (2017) and reported high as compared to the findings of Gradiz *et al*. (2009) and Cankaya *et al*. (2011). The persistency of lactation found in GTF was also higher than that found for Holstein cattle (Tekerli *et al*., 2000; Rekik *et al*., 2006; Atashi *et al*., 2007). Existing differences in these parameters might be the result of a combination of genetic, diet, management and specific climatic effects. The positive parameter 'a' in all models clearly indicated that this parameter explained the increasing part of the lactation curve. Based on the sign/direction of the parameters 'b' and 'c' obtained in the present study, it was determined that IPM curve followed by GTF which applied to fit the 305 days milk data were typical standard curve for crossbred dairy cattle.

Relationship between lactation curve parameters

The Estimates of phenotypic correlation among lactation curve parameters in crossbred cattle was presented in Table 3.

Table 2: The estimates of average lactation curve parameters of crossbred cattle under various non-linear models.

Models	Lactation curve parameters				
	a	b	c	d	f
EDF	16.34±0.11		$0.0018\pm4.3\times10^{-5}$		
PEM	15.29±0.14	-0.0004±0.0002	$4.84\times10^{-6}\pm4.81\times10^{-7}$		
IPM	0.29±0.007	0.0484±0.0003	$0.000194\pm1.82\times10^{-6}$		
GTF	10.18±0.28	0.141±0.008	$0.0031\pm7.4\times10^{-5}$		
MLF	7.21±0.29	-1.444±0.025	4.668±0.120		
PRM	5.11±0.28	-0.097±0.002	$0.00013\pm4.742\times10^{-6}$	4.240±0.099	-6.523±0.152

For GTF PY: Peak Yield (15.15), DIMP = days in milk at peak production (45.48), and P: Persistency (6.59).

Table 3: Estimates of correlation among lactation curve parameters in crossbred dairy cattle.

	a	b	c	d	f
Exponential Decline Function					
a			0.837**		
Parabolic Exponential Model (above diagonal) and Inverse Polynomial Model (below diagonal)					
a		-0.874**	-0.755**		
b	-0.811**		0.965**		
c	0.641**	-0.890**			
Gamma-Type Function (above diagonal) and Mixed Log Function (below diagonal)					
a		-0.981**	-0.833**		
b	0.885**		0.918**		
c	-0.967**	-0.973**			
Polynomial Regression Function					
a		0.886**	-0.757**	0.980**	0.969**
b			-0.977**	0.937**	-0.967**
c				-0.854**	0.909**
d					0.992**

** P < 0.001.

The results of the present study clearly indicated that the relationships between parameters b and c for IPM, MLF and PRM were highly significant (P<0.01), negative and high with corresponding values of -0.890, -0.973 and -0.977, in that order while the correlation between b and c parameters in the PEM (0.965) and GTF (0.918) models were positive, highly significant (P<0.01) and higher. For the PRM, the values of correlation between b and f and c and d parameters were negative (P<0.01). Parameters a and b were associated negatively and significantly (P<0.01) for PEM (-0.874), IPM (-0.811) and GTF (-0.981) whereas correlation between a and b parameters of PRM and MLF were highly significant (P<0.01), positive and high with respective values of 0.886 and 0.885, respectively. The parameters a and c were associated negatively (P<0.01) for PEM (-0.755), MLF (-0.967), GTF (-0.833), and PRM (-0.757) whereas correlation between a and c parameters of EDF, and IPM were positive (P<0.01). The relationships among all the lactation curve parameters are important, specially between b and c parameters because (a) is always positive and influence the average level of production (Ali and Schaeffer 1987). The negative relationship between b and c observed in the IPM, MLF and PRM models clearly demonstrated that high daily milk production may be maintained throughout the lactation which in turn has high implication for economic return of the dairy producers; but the positive relationships between b and c found for PEM and GTF low persistency hence high milk production may not be maintained. The negative correlation between a and b parameters found in the present study for the PEM, IPM and GTF clearly showed that the crossbred cows with smaller initial daily milk production might have high peak milk yield. However, the positive association between parameters a and b observed in our study for the PRM and MLF noticeably indicated that the crossbred cows with high initial daily milk production would have low peak milk production; but average milk yield over the complete lactation could be high in both cases.

Fitting of lactation curves with daily milk yield

Lactation curves of observed versus predicted 305 days daily milk yield (Kg) have been presented separately (Fig 1-6) to illustrate the fitness of all the models used in the study for crossbred dairy cattle. The Polynomial regression model gave highest fit to the daily milk yield data (Fig 6; Table 4) with R^2, MSE, AIC and CV values of 98.10%, 0.087, -743.31 and 2.37 %, respectively. The Inverse Polynomial Model have also best fitted the observed daily milk yield data with predicted daily milk yield data (Fig 1) with highest R^2 (98.05%), lower MSE (0.089), low AIC (-735.8972) and lower CV (2.40%) values. The fitting of observed daily milk yield data with predicted ones were also found to be higher in the Mixed Log Function (Fig 3), and Gamma-Type Function (Fig 2). The R^2, MSE, AIC and CV estimates observed for Mixed Log Function and Gamma-Type Function were 96.46%,

Table 4: Estimates of different parameters for goodness of fit for various nonlinear models.

Models	Estimates of parameters for goodness of fit						Ranks
	R^2	R^2Adj	AIC	MSE	SSE	CV (%)	
EDF	0.8602	0.8598	-183.6897	0.54400	164.83	5.93	6
PEM	0.8961	0.8957	-253.6704	0.43247	131.03	5.29	5
IPM	0.9805	0.9804	-735.8972	0.08898	26.96	2.39	2
GTF	0.9585	0.9584	-505.2358	0.18956	57.44	3.50	4
MLF	0.9646	0.9645	-558.16	0.15936	48.29	3.20	3
PRM	0.9810	0.9810	-743.31	0.08685	26.31	2.36	1

R^2: coefficient of determination, AIC: Akaike information criterion, MSE: Mean Square Error, SSE: Sum Square Error, CV: Coefficient of variation.

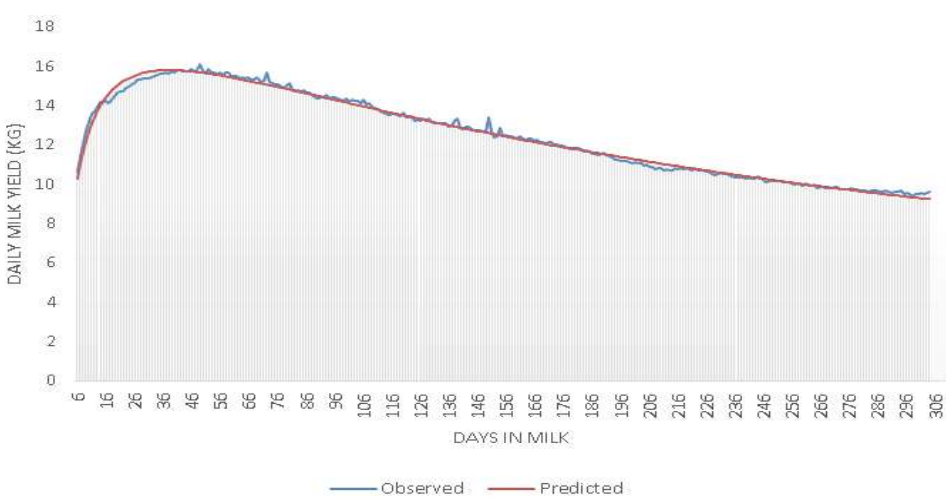

Fig 1: Lactation curves of observed versus predicted daily milk yield (Kg) over 305 milk days for Inverse Polynomial Model.

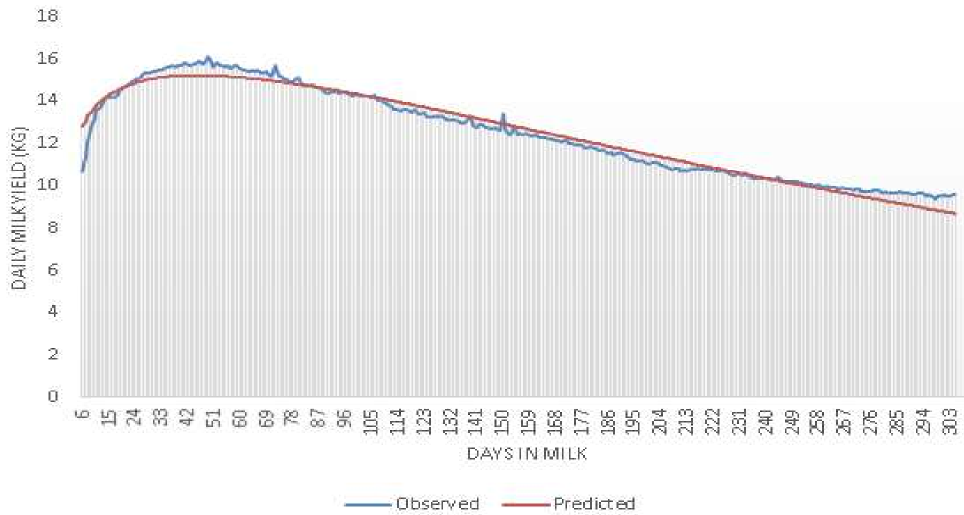

Fig 2: Lactation curves of observed versus predicted daily milk yield (Kg) over 305 milk days fitted under Gamma-type Function.

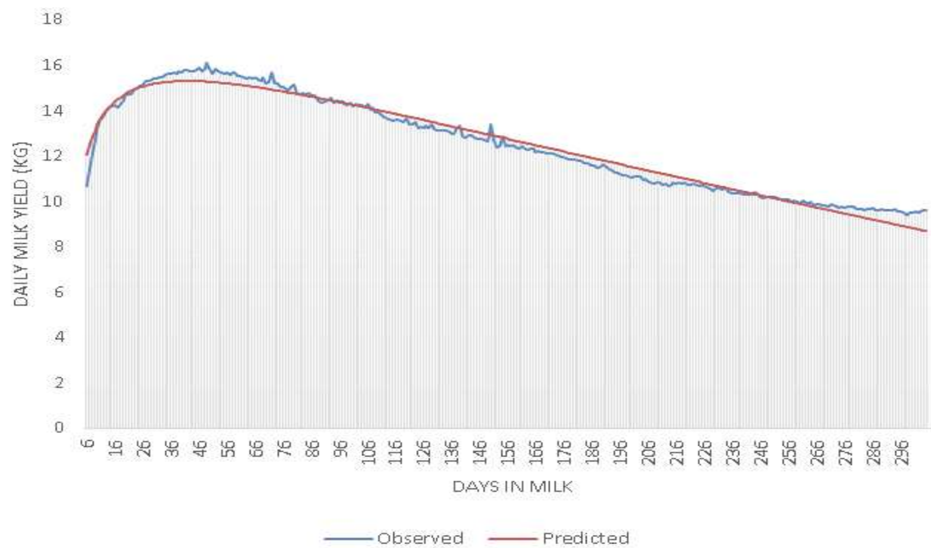

Fig 3: Lactation curves of observed versus predicted daily milk yield (Kg) over 305 milk days fitted under Mixed Log Function.

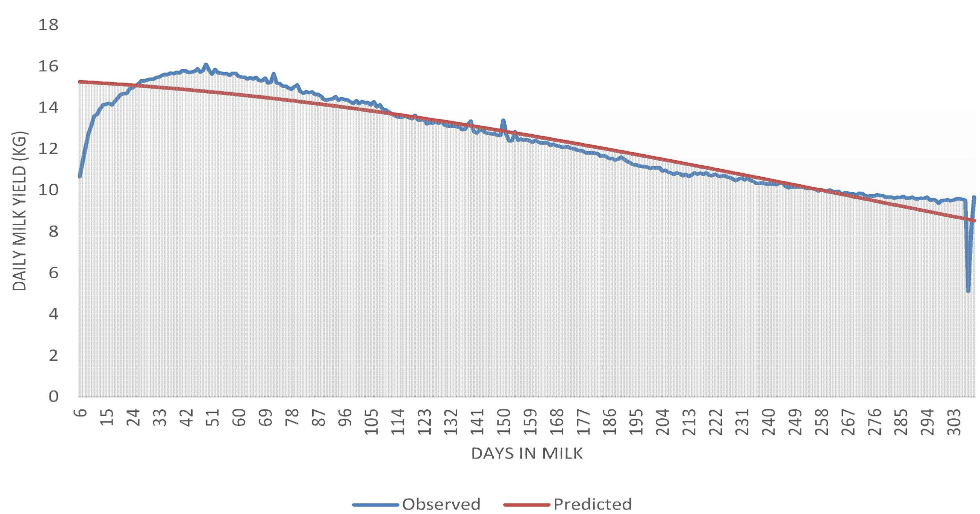

Fig 4: Lactation curves of observed versus predicted daily milk yield (Kg) over 305 milk days fitted under Parabolic Exponential Model.

0.159, -558.16 and 3.21%; 95.85%,0.190,-505.24 and 3.50%, respectively. However, the Exponential Decline Function and Parabolic Exponential Model depicted relatively low fit to the daily milk yield data when compared with the other non-linear models used in the study. The R^2, MSE, AIC and CV values for Exponential Decline Function were 86.02%, 0.54, -183.69, 5.94%, respectively whereas the Parabolic Exponential Model described the milk yield data of crossbred dairy cattle with R^2, MSE, AIC and CV estimates of 89.61%, 5.29, -253.67 and 0.43%, respectively. Coefficients of determination (R^2) values have been used to evaluate the fit of the models in some studies (Akbaş *et al*., 2006). The models that gave the highest R^2 values have been accepted as the best fitting models. Based on the values of model parameters investigated in the present study, nonlinear models namely EDF and PEM could adequately fit the daily milk yield data for 305 days lactation; while PRM, and IPM models followed by MLF and GTF models gave best fit and reliable description to the lactation curve pattern and characteristics of crossbred cattle. However only two models namely IPM and GTF could be chosen with corresponding developed equations for predicting daily milk production from calving to 305 days in milk of crossbred dairy cattle managed under DLF of GADVASU because IPM and GTF curves were standard typical curves for the cattle. Singh *et al*. (1998) reported that Inverse Quadratic Polynomial (IQP) model was the best function in explaining the first lactation curve based on monthly as well as weekly milk records of Jersey x Sahiwal F1 cows whereas Yogesh *et al*. (2017) for Gir crossbreds with R^2=90.50% and Cole and Null (2009) for dairy cattle with R^2=91.25% under Gamma Type and Tekerli *et al*. (2000) reported that Log Transformed Gamma Function gave best fit to daily milk yield of Holstein cows with R^2 = 70.80%.

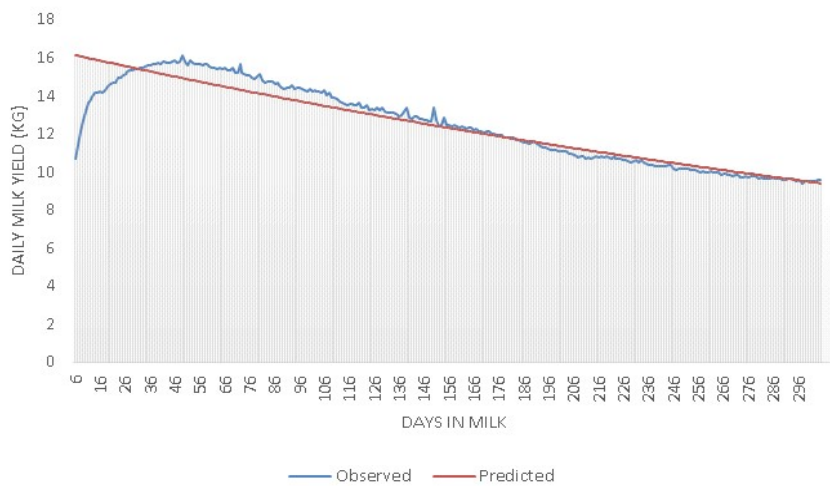

Fig 5: Lactation curves of observed versus predicted daily milk yield (Kg) over 305 milk days fitted under Exponential decline function.

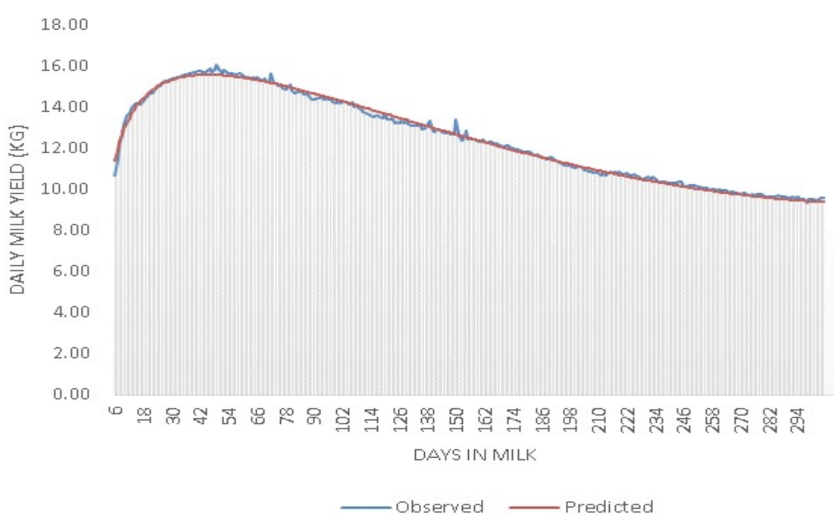

Fig 6: Lactation curves of observed versus predicted daily milk yield (Kg) over 305 milk days fitted under Polynomial Regression Function.

Conclusion

It was concluded that the PRM and IPM followed by MLF and GTF models depicted best fit to the data of daily milk yield within 6 to 305 days in milk and reliable description to the lactation curve pattern and characteristics of crossbred dairy cattle. However only two models, namely IPM and GTF could be chosen and recommended for accurate prediction of daily milk yield in crossbred population, because IPM and GTF curves were standard typical curves for the cattle. Also, the lactation parameters estimated through this study can be used for genetic evaluations and selection of cows to improve persistency and quantum of milk yield.

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