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NONLINEAR MODELS FOR POULTRY EGG PRODUCTION

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ABSTRACT

Two nonlinear models were proposed to describe the poultry egg production curves for first year egg production. These models were compared with gamma type, McNally modified gamma type, compartment, Adams-Bell model, double compartment, logistic – curvilinear and logistic – linear on the basis of two data sets pertaining to egg production records arranged in 4-weekly intervals. The fitting of these models was judged with coefficient of determination, mean square error and mean absolute error. A proposed model expressed as $N_t = a\, t^b \exp(ct^2 + d\text{Ö}\, t)$ was found best among all the models with respect to one data set and described egg production curve better than gamma type, compartment model, Adams-Bell model, double compartment model and McNally model for both data sets.

Key words: Nonlinear models, Poultry Egg production.

INTRODUCTION

The annual poultry egg production summarized on weekly, biweekly or monthly basis increases rapidly to a peak and gradually decreases and reaches to its some minimum value. The mathematical models used to describe the egg production curves are useful for many management decisions to be taken to increase egg production and to predict total egg production on the basis of partial records. Several nonlinear models have been proposed for poultry egg production by different researchers (Compartment model by McMillan *et al*, 1970; modified gamma type by McNally, 1971; function as a difference of increasing function and linear function by Adams and Bell, 1980; double compartment model as a difference of two exponential functions by McMillan, 1981;logistic - curvilinear by Cason and Britton, 1988 and modified compartment model by Yang *et al.*, 1989). Cason and Ware (1990) generated generalized egg production models by combining growth models as increasing terms and linear, corrected linear and curvilinear functions as decreasing terms. Prasad and Singh (2007) proposed two alternative forms of logistic – curvilinear model. Adams and Bell reported that the fit of McNally model to percent hen-day production was not as good after the peak as their model. So, in the present study two nonlinear models were proposed as modified form of McNally's model and were compared with some existing models by using two data sets on 4-weekly egg production of laying hens.

MATERIAL AND METHODS

Model Development

McNally (1971) observed the similarity between chronological changes in annual egg production of poultry and milk production for dairy cattle during a lactation and used incomplete Gamma type function

$$M_1 : N_t = a\, t^b e^{-ct} + \varepsilon_t$$

proposed by Wood (1967) for milk production during a lactation and suggested its following modified form

$$M_2 : N_t = a\, t^b e^{-ct + d\text{Ö}t} + \varepsilon_t$$

for egg production of chickens. Following two nonlinear models are proposed by changing functional form of above model M_2 to describe egg production curve.

$$M_3 : N_t = a\, t^b e^{-ct + d/\text{Ö}t} + \varepsilon_t$$

$$M_4 : N_t = a\, t^b e^{ct^2 + d\text{Ö}t} + \varepsilon_t$$

These above models were compared with following existing models

Compartment model (McMillan *et al.*, 1970)

$$M_5 : N_t = a\, e^{-bt} (1 - c\, e^{-dt}) + \varepsilon_t$$

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Table 1: Estimated parameters of different models and measures of goodness-of-fit

Model	Estimated parameters with standard errors					Measures			
	a	b	c	d	e	R^2(%)	MSE	MAE	MF
		SET 1							
M_1	10.1399	0.9229	0.1727	-	-	77.36	3.733	1.415	10.
	(1.4644)	(0.1915)	(0.0344)						
M_2	3895.9623	4.6989	-0.6146	-7.1339	-	90.91	1.666	0.863	6.1
	(6671.1610)	(1.1363)	(0.2264)	(2.0770)					
M_3	38387.505	-2.4308	-.0544	-8.8929	-	89.46	1.932	0.938	6.6
	(105905.8)	(1.1100)	(0.0763)	(3.0077)					
M_4	220.6653	3.3615	0.0096	-3.6164	-	91.10	1.631	0.868	6.0
	(141.6953)	(0.6420)	(0.0034)	(0.7732)					
M_5	26.0519	0.0560	1.5434	0.7172	-	88.52	2.104	0.983	6.4
	(3.8600)	(0.0156)	(0.4050)	(0.2634)					
M_6	0.3865	-0.7074	0.8414	76.5959	-	88.13	2.175	1.007	6.5
	(0.1021)	(0.4410)	(0.2086)	(2.0080)					
M_7	33.1974	0.0759	0.4191	-	-	80.74	3.176	1..324	9.9
	(12.7341)	(0.0330)	(0.1483)						
M_8	24.9120	0.0521	11.8785	1.4576	-	90.86	1.675	0.777	5.0
	(2.1435)	(0.0101)	(5.9694)	(0.3199)					
M_9	18.7007	34.9082	1.9439	0.7878	4.1403	90.22	2.016	0.810	5.0
	(7.5628)	(113.2713)	(1.3140)	(0.1762)	(6.6928)				
		SET II							
M_1	9.1982	0.9521	0.1632	-	-	83.28	2.615	1.248	8.1
	(1.1853)	(0.1648)	(0.0286						
M_2	353.3790	3.3130	-0.3161	-4.2966	-	88.46	2.060	1.000	7.3
	(649.5620)	(1.2166)	(0.2422)	(2.2242)					
M_3	892.8011	-0.8921	0.0410	-4.9506	-	87.10	2.242	1.085	7.7
	(2548.5887)	(1.1532)	(0.0797)	(3.0983)					
M_4	85.7967	2.6849	0.0053	-2.6611	-	88.86	1.937	0.965	7.0
	(59.3477)	(0.6941)	(0.0037)	(0.8329)					
M_5	28.5174	0.0575	1.2158	0.4699	-	87.41	2.189	1.067	7.3
	(7.2919)	(0.0234)	(0.2456)	(0.2300)					
M_6	0.5084	-1.3160	0.8585	75.4277	-	87.59	2.140	0.720	7.3
	(0.0982)	(0.5547)	(0.2749)	(2.8486)					
M_7	37.5744	0.0778	0.3427	-	--	84.77	2.383	1.210	8.2
	(18.9536)	(0.0329)	(0.1287)						
M_8	25.1948	0.0472	8.6582	1.1492	-	91.73	1.439	0.881	6.1
	(2.3766)	(0.0104)	(3.3333)	(0.2316)					
M_9	15.7527	132.1138	2.1349	0.6921	6.9507	93.45	1.281	0.711	4.3
	(2.5195)	(256.6180)	(0.7622)	(0.1386)	(1.9429)				

The figures in parentheses represent the asymptotic standard error of the parameters

Adams-Bell model (Adams and Bell, 1980)

$$M_6: N_t = [100/(1 + a^{t-b})] - ct + d + \varepsilon_t$$

Double compartment model (McMillan, 1981)

$$M_7: N_t = a(e^{-bt} - e^{-ct}) + \varepsilon_t$$

Logistic curvilinear (Cason and Britton, 1988)

$$M_8: N_t = a e^{-bt}/(1 + c e^{-dt}) + \varepsilon_t$$

Logistic - linear (Cason and Ware, 1990)

$$M_9: N_t = [a/(1 + b e^{-ct})] - (dt - e) + \varepsilon_t$$

In all the above models, N_t is the number of eggs produced in time t and ε_t is the error term associated with N_t.

Model comparison

All these models were fitted to annual egg production summarized in 28-day intervals from 20-th to 72-th week of age of Aseel x CARI Red and CARI Red x Aseel maintained at Central Avian Research Institute, Izatnagar (U.P.). Since all the above models are nonlinear, the fitting of the models was done by using nonlinear optimization technique

Table 2: F- test statistics for model comparison

Model	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9
M_1	1	13.412**	10.327*	13.888**	8.742*	8.163*	1.176	13.287**	5.259*
M_2	4.043	1	1.160	1.021	1.263	1.306	10.065*	1.006	0.564
M_3	2.665	1.118	1	1.184	1.089	1.126	7.441*	1.153	0.623
M_4	4.506	1.036	1.158	1	1.290	1.334	10.470*	1.027	0.718
M_5	2.951	1.091	1.024	1.130	1	1.034	6.092*	1.256	1.394
M_6	3.224	1.067	1.048	1.105	1.023	1	5.601*	1.298	1.710
M_7	1.098	2.884	1.628	3.250	1.889	2.138	1	9.958*	3.876
M_8	9.183*	1.394	1.559	1.346	1.521	1.487	7.567*	1	0.522
M_9	6.211*	6.029*	7.757*	5.609*	7.380*	7.037*	5.304*	2.109	1

Note : 1.The entries above and below the leading diagonal correspond to Set I and Set II respectively.
2. * Significant ($P < 0.05$) and ** Significant ($P < 0.01$)

Table 3: Summary measures of different estimated egg productions curves

Summarymeasures	Models								
	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9
					SET I				
FPP (3.67)	8.53	5.75	5.57	5.99	6.07	6.09	8.94	6.28	6.44
PP (5)	5	4	4	4	4	4	5	4	4
PEP (22.00)	18.88	19.54	19.23	19.34	19.00	18.91	18.63	19.54	19.39
LPP (14.87)	11.45	13.31	12.95	13.53	12.57	12.47	12.23	12.65	12.57
	SET II								
FPP (6.43)	7.81	5.97	6.07	6.05	6.46	6.45	8.09	6.42	7.21
PP (5)	6	5	5	5	5	5	6	5	4
PEP (20.85)	19.03	19.09	18.91	19.20	18.91	18.90	18.75	19.36	19.54
LPP (12.10)	12.67	13.77	13.47	13.95	13.47	13.41	13.22	13.64	13.71

Note: The figures in parentheses in first column are the observed values.
FPP : First period production of eggs
PP : Peak period
PEP : Peak egg production
LPP : Last period production of eggs

(Levenberg- Marquardt) with the help of SPSS, version-12. The goodness – of – fit of the models was judged with coefficient of determination (R^2), mean square error (MSE), mean absolute error (MAE) and mean absolute percent error (MAPE). The statistical significance between models in terms of the goodness - of - fit was assessed using an F test. For models with the same number of parameters the test statistic $F = SS_1 / SS_2$ was used where SS_1 and SS_2 are the values of the residual sum of squares corresponding to model having bigger and smaller error sum of squares respectively. For comparing two models with unequal number of parameters, following F test statistic was used

$$F = [(SS_1 - SS_2) / (df_1 - df_2)] / (SS_2 / df_2)$$

where df is the degrees of freedom and subscript 1 refers to model with fewer parameters (Motulskey and Ransnas, 1987). To study the nature of errors i.e. randomness and normality, Run test & serial correlation of first order and Shapiro-Wilk test were used and their values were calculated with the help of SPSS package.

RESULTS AND DISCUSSION

The estimated parameters along with their standard errors and measures of goodness – of – fit are given in Table 1. The smallest value of MSE, MAE and MAPE and highest value of R^2 correspond to model M_4 and M_9 with respect to data set I and II respectively and implying their best fitting to respective data sets. The next models in order of merit of fitting might be considered as M_2 and M_8 & M_8 & M_4 for these respective data sets with marginal differences of these measures. Thus, the proposed model M_4 has shown improved fitting to these two data sets over M_1, M_2 and M_3 models. From Table 2 showing the F test statistics for testing the significance difference between any two models, it is revealed that there was no significant difference among

Table 4: Tests for randomness and normality

Particulars	Models								
	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9
	Test for Randomness - Run Test								
Set I									
No. of Runs	4	8	6	8	6	6	5	6	7
Z-value	-1.727	0.213	-0.402	0.213	-0.561	-0.561	-1.144	-0.402	0.000
Significance	0.084	0.831	0.688	0.831	0.575	0.575	0.253	0.688	1.000
Set II									
No. of Runs	5	6	6	8	6	6	5	8	8
Z-value	-1.107	-0.561	-0.561	0.022	-0.561	-0.561	-1.144	0.231	0.022
Significance	0.309	0.575	0.575	0.982	0.575	0.575	0.253	0.831	0.982
	Test for Randomness – Serial Correlation								
Set I	0.539	0.112	0.161	0.108	0.276	0.311	0.464	0.265	0.295
Set II	0.348	-0.132	-0.045	-0.154	0.004	0.016	0.265	-0.059	0.025
	Test for Normality – Shapiro-Wilk Test								
Set I									
Statistics	0.927	0.963	0.973	0.957	0.975	0.977	0.944	0.952	0.958
Significance	0.311	0.803	0.923	0.709	0.948	0.965	0.506	0.636	0.719
Set II									
Statistics	0.900	0.945	0964	0.938	0.972	0.970	0.939	0.889	0.921
Significance	0.133	0.522	0.812	0.428	0.916	0.890	0.442	0.094	0.260

models having four or five parameters with respect to goodness-of-fit. All the four or five parameter models fitted data set I significantly ($P < 0.05$) better than M_1 and M_7. Gavora *et al.* (1982) and McMillan *et al.* (1986) reported poor fitting of Wood model as compared to compartment model. The model M_9 has been observed to be significantly ($P < 0.05$) better than all the models except M_8 with respect to fitting of data set II whereas M_8 performed significantly ($P < 0.05$) better than M_1 and M_7.

Table 3 presents four summary measures viz. first period production of eggs (FPP), peak period (PP), peak egg production (PEP) and last period production of eggs (LPP) of different estimated egg production curves along with auto-correlation of first order among errors generated by different models. The three - parameter models (M_1 and M_7) estimated higher values of FPP as compared to other models for both data sets. All the four and five parameter models underestimated peak period by one period for data set 1. However, models M_1 and M_7 overestimated PP by 1 period for data set 2. All the models underestimated PEP (1 to 2 eggs) for both data sets and LPP for data set I only. The estimated values of LPP by different models are approximately equal to observed values for data set II. Thus, no model estimated all the summary measures equal to observed values, the reason might be the nature of collected data i.e. non-smoothness of the observed egg production curves.

Table 4 presents the test statistics for testing the randomness of errors and normality behaviour. The first order serial correlation coefficients among errors for different models are observed to be non-significant confirming the independence of the errors. The Run test statistics for both data sets I and II are found significant at different probability levels (greater than $p = 0.05$) which shows that the errors generated from different models are randomly distributed. Similarly, the Shapiro-Wilk test statistics are also significant at different probability levels greater than 0.05, which confirms the normal distribution of errors. Thus, the errors are randomly and normally distributed variables. Therefore, on the basis of above observations, it could be concluded that the proposed model M_4 may be used in place of gamma type, compartment model, Adams-Bell model, double compartment model and McNally model for better estimation of egg production curves of laying hens.

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